Chapter → 1 Algebra

8/12/16

$$a^3+b^3 \Rightarrow (a+b)(a^2+b^2-ab)$$
$$a^3-b^3 \Rightarrow (a-b)(a^2+b^2+ab)$$

Algebra

Type-1

If $x+\frac{1}{x}=K$, then $x^2+\frac{1}{x^2}=K^2-2$

$x-\frac{1}{x}=K$, then $x^2+\frac{1}{x^2}=K^2+2$

Question

Maths by Puneet chaudhary sir

① If $x+\frac{1}{x}=4$, $x^2+\frac{1}{x^2}=?$

Solve

$$x^2+\frac{1}{x^2}=K^2+2 \Rightarrow 4^2-2=16-2=14$$

② If $x+\frac{1}{x}=5$, $x^2+\frac{1}{x^2}=?$

Solve

$$x^2+\frac{1}{x^2}=K^2-2 \Rightarrow 5^2-2 \Rightarrow 25-2 \Rightarrow 23$$

③ If $x-\frac{1}{x}=5$, $x^2+\frac{1}{x^2}=?$

Solve

$$x^2+\frac{1}{x^2}=K^2+2 \Rightarrow 5^2+2=25+2=27$$

④ If $x-\frac{1}{x}=10$, $x^2+\frac{1}{x^2}=?$

Solve

$$x^2+\frac{1}{x^2}=K^2+2=10^2+2=100+2=102 //$$

⑤ If $x-\frac{1}{x}=11$, $x^2+\frac{1}{x^2}=?$

Solve

$$x^2+\frac{1}{x^2}=K^2+2 \Rightarrow 11^2+2 \Rightarrow 121+2=123 //$$

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If $x+\frac{1}{x}=\sqrt{2}$

$\Rightarrow \boxed{x^2+\frac{1}{x^2}=0}$ $\boxed{x^4=-1}$, $(-1)^{odd}=-1$

$(-1)^{even}=+1$

Q-5 If $x+\frac{1}{x}=\sqrt{2}$, $x^{16}+x^{12}+x^{8}+x^{4}+1=?$

Solve $x^{16}+x^{12}+x^{8}+x^{4}+1 \Rightarrow (x^4)^4+(x^4)^3+(x^4)^2+x^4+1$

$[\because x^4=-1]$ $\Rightarrow (-1)^4+(-1)^3+(-1)^2\cancel{-1}\cancel{+1}$

$\Rightarrow 1-\cancel{1}\cancel{+1} \Rightarrow 1$ Ans

Q-6 if $x+\frac{1}{x}=\sqrt{2}$, $x^{60}+\frac{1}{x^{60}}=?$

Solve $(x^4)^{15}+\frac{1}{(x^4)^{15}} \Rightarrow (-1)^{15}+\frac{1}{(-1)^{15}} \Rightarrow -1-1 \Rightarrow -2$ Ans

Q-7 if $x+\frac{1}{x}=\sqrt{2}$, $x^{61}+\frac{1}{x^{61}}=?$

Solve $x^{61}+\frac{1}{x^{61}} \Rightarrow x\cdot(x^4)^{15}+\frac{1}{x\cdot(x^4)^{15}} \Rightarrow x(-1)+\frac{1}{x(-1)}$

$\Rightarrow -x-\frac{1}{x}$

$\Rightarrow -\left(x+\frac{1}{x}\right) \Rightarrow -\sqrt{2}$ Ans

Q.8 If $x+\frac{1}{x}=\sqrt{2}$, $x^{99}+x^{95}+1=?$

Solve $x^{99}+x^{95}+1 \Rightarrow x^{95}(x^4+1)+1$

$\Rightarrow x^{95}(-1+1)+1$

$\Rightarrow 0+1 \Rightarrow 1$ Ans

If $x+\frac{1}{x}=K$, $x^3+\frac{1}{x^3}=K^3-3K$

$x-\frac{1}{x}=K$, $x^3-\frac{1}{x^3}=K^3+3K$

Question

Q-9 If $x+\frac{1}{x}=3$, $x^3+\frac{1}{x^3}=?$

Solve $x^3+\frac{1}{x^3}=K^3-3K \Rightarrow (3)^2-3\times3 \Rightarrow 27-9 \Rightarrow 18$ Ans

Q-10 If $x-\frac{1}{x}=4$, $x^3-\frac{1}{x^3}=?$

Solve $x^3-\frac{1}{x^3}=K^3-3K \Rightarrow (4)^3+3\times4 \Rightarrow 64+12 \Rightarrow 76$ Ans

If $x+\frac{1}{x}=\sqrt{3}$

$\Rightarrow \boxed{x^3+\frac{1}{x^3}=0}$ $\boxed{x^6=-1}$

$\boxed{x^4+x^0=0}$

$\boxed{x^6+x^0=0}$

Q-11 If $x+\frac{1}{x}=\sqrt{3}$, $x^{18}+x^{12}+x^6+1=?$

Solve $x^{18}+x^{12}+x^6+1 \Rightarrow (x^6)^3+(x^6)^2+x^6+1 \Rightarrow (-1)^3+(-1)^2-1+1$

$\Rightarrow \cancel{-1}\cancel{+1}\cancel{-1}\cancel{+1}$

$\Rightarrow 0$ Ans

Q-12 If $x^{60} + \frac{1}{x^{60}} = ?$ $\qquad x + \frac{1}{x} = \sqrt{3}$

Solve

$$x^{60} + \frac{1}{x^{60}} \Rightarrow (x^6)^{10} + \frac{1}{(x^6)^{10}} \Rightarrow (-1)^{10} + \frac{1}{(-1)^{10}}$$

$$\Rightarrow 1 + 1 \Rightarrow 2 \ \underline{\text{Ans}}$$

Q-13 If $x + \frac{1}{x} = \sqrt{3}$, $x^{61} + \frac{1}{x^{61}} = ?$

Solve

$$x.(x^6)^{10} + \frac{1}{x.(x^6)^{10}} \Rightarrow x(-1)^{10} + \frac{1}{x(-1)^{10}}$$

$$\Rightarrow x + \frac{1}{x} \Rightarrow \sqrt{3} \ \underline{\text{Ans}}$$

Q-14 If $x^{99} + x^{93} + 1 = ?$ $\qquad x + \frac{1}{x} = \sqrt{3}$

Solve

$$x^{99} + x^{93} + 1 \Rightarrow x^{93}(x^6 + 1) + 1 \Rightarrow x^{93}(-1+1) + 1$$

$$\Rightarrow 0 + 1 = 1 \ \underline{\text{Ans}}$$

Type → III

If $x + \frac{1}{x} = K$, $x - \frac{1}{x} = \sqrt{K^2 + 4}$

$x - \frac{1}{x} = K$, $x + \frac{1}{x} = \sqrt{K^2 + 4}$

Q-15 If $x + \frac{1}{x} = 3$, $x - \frac{1}{x} = ?$

Solve

$$x - \frac{1}{x} = \sqrt{K^2 - 4} \Rightarrow \sqrt{3^2 - 4} \Rightarrow \sqrt{9 - 4} \Rightarrow \sqrt{5}$$

Q-16 If $x+\frac{1}{x}=5$, $x-\frac{1}{x}=?$

Solve ⇒ $x-\frac{1}{x}=\sqrt{5^2-4}=\sqrt{25-4}=\sqrt{21}$ Ans

Q.17 If $x-\frac{1}{x}=5$, $x+\frac{1}{x}=?$

Solve

$x+\frac{1}{x}=\sqrt{K^2+4} \Rightarrow \sqrt{5^2+4}=\sqrt{29}$ Ans

Q.18 If $x-\frac{1}{x}=6$, $x+\frac{1}{x}=?$

Solve $x+\frac{1}{x}=\sqrt{K^2+4}=\sqrt{36+4}=\sqrt{40}=2\sqrt{10}$ Ans

Concept

If

$x+\frac{1}{x}=3$, $x-\frac{1}{x}=?$

$x^2+\frac{1}{x^2}=K^2-2=$ $x^2+\frac{1}{x^2}-2=7-2$

$\left(x^2+\frac{1}{x}\right)^2=5$

$$\boxed{x-\frac{1}{x}=\sqrt{5}}$$

Q-19 If $x+\frac{1}{x}=3$, $x=?$

Solve

$x+\frac{1}{x}=3$

$x-\frac{1}{x}=\sqrt{5}$

$2x=3+\sqrt{5}$

$x=\frac{3+\sqrt{5}}{2}$ Ans

Trick

$x+\frac{1}{x}=K$, $x=?$

$x=\frac{K+\sqrt{K^2-4}}{2}$

Q-20 If $x+\frac{1}{x}=5$, $x=?$

Solve

$$x=\frac{K+\sqrt{K^2-4}}{2}=\frac{5+\sqrt{21}}{2}$$ Ans

Q·21 If $x-\frac{1}{x}=3$, $x=?$

$$x-\cancel{\frac{1}{x}}=3$$

$$x+\cancel{\frac{1}{x}}=\sqrt{13}$$

$$2x=3+\sqrt{13}$$

$$\boxed{x=\frac{3+\sqrt{13}}{2}}$$

Trick

$x-\frac{1}{x}=K$, $x=?$

$$x=\frac{K+\sqrt{K^2+4}}{2}$$

Q-22 If $x-\frac{1}{x}=4$, $x=?$

Solve

$$x=\frac{4+\sqrt{4^2+4}}{2}\Rightarrow\frac{4+\sqrt{20}}{2}\Rightarrow\frac{4+2\sqrt{5}}{2}$$ Ans

Roots

Type → 2

① $x^a \times x^b \Rightarrow x^{a+b}$

② $\frac{x^a}{x^b} \Rightarrow x^{a-b}$

③ $\sqrt{x}\ ,\ \sqrt[3]{y}\ ,\ \sqrt[4]{z}$

$\downarrow \quad \downarrow \quad \downarrow$

$x^{1/2} \quad y^{1/3} \quad z^{1/4}$

④ $\sqrt{\sqrt{x}}\ ,\ \sqrt{\sqrt[3]{y}}\ ,\ \sqrt[3]{\sqrt[4]{z}}$

$\downarrow \quad \Downarrow \quad \Downarrow$

$\sqrt[4]{x}\ ,\ y^{1/6}\ ,\ z^{1/12}$

$\Downarrow$

$x^{1/4}$

Inverse

⑤ $a^{-1} = \frac{1}{a}$

⑥ $x^a = y^b\ ,\ x = ?$

$x = y^{b/a}$

⑦ x^{a^b}

eg. ⇒ x^{2^3}

$x^{2\times2\times2} \Rightarrow x^8$ Ans

⑧ $((x)^a)^b$ eg. ⇒ $((x)^2)^3 \Rightarrow x^6$ Ans

⑨ $x^a = x^b$.

then $a = b$

eg. ⇒ $x^a = x^{400}$

$$\boxed{a = 400}$$

Type → 1

① $x = \sqrt{17\sqrt{17\sqrt{17\ldots\infty}}}$

Solve

Concept ⇒ $x = \sqrt{17\sqrt{17\sqrt{17\ldots\infty}}}$

Sq. करने पर

$x^2 = 17\sqrt{17\sqrt{17\ldots\infty}}$

$(\sqrt{17\sqrt{17\ldots\infty}} = x)$

$x^{\cancel{2}} = 17.\cancel{x}$

$\boxed{x = 17}$

Note :→ इस प्रकार के प्रश्नों में उत्तर वही होता है जो संख्या √ के अंदर होती है।

② $x = \sqrt{3\sqrt{3\sqrt{3\ldots\infty}}}$

Solve ⇒ $x = 3$

③ $x = \sqrt{5\sqrt{5\sqrt{5\ldots\infty}}}$

Solve ⇒ $x = 5$

④ $x = \sqrt[3]{17\sqrt[3]{17\sqrt[3]{17\ldots\infty}}}$

Solve

Concept ⇒ $x = \sqrt[3]{17\sqrt[3]{17\sqrt[3]{17\ldots\infty}}}$

Cube करने पर

$x^3 = 17\sqrt[3]{17\sqrt[3]{17\ldots\infty}}$

$(\sqrt[3]{17\sqrt[3]{17\ldots\infty}} = x)$

$x^3 = 17.\cancel{x}$

$\boxed{x = \sqrt{17}}$

Note :→ इस प्रकार के प्रश्नों में उत्तर √ के अंदर वही आता है जो संख्या ∛ के अंदर होती है।

⑤ $x = \sqrt[3]{x\sqrt[3]{x\ldots}}\infty$

Solve

$$x = \sqrt{x} \quad \text{Ans}$$

⑥ $x = \sqrt[3]{4\sqrt[3]{4\ldots}}\infty$

Solve

$$x = \sqrt{4} \quad \text{Ans}$$

⑦ $x = \sqrt[3]{5\sqrt[3]{5\ldots}}\infty$

Solve

$$x = \sqrt{5} \quad \text{Ans}$$

Imp

⑧ $x = \sqrt{a\sqrt[3]{b\sqrt{a\sqrt[3]{b\ldots}}}}\infty$

Solve

$$x = \sqrt{a\sqrt[3]{b\cdot x}} \qquad \left(x = \sqrt{a\sqrt[3]{b}\ldots\infty}\right)$$

Sq. करने पर

$$x^2 = a\sqrt[3]{b\cdot x}$$

Cube करने पर

$$x^6 = a^3 b\cdot \cancel{x}$$

$$x^5 = a^3 b$$

$$\boxed{x = (a^3 b)^{1/5}} \quad \text{Ans}$$

Imp

⑨ $x = \sqrt{3\sqrt[3]{2\sqrt{3\sqrt[3]{2\ldots}}}}\infty$

Solve

$$x = \sqrt{3\sqrt[3]{2\cdot x}} \qquad \left(x = \sqrt{3\sqrt[3]{2\ldots}\infty}\right)$$

Sq. करने पर

$$x^2 = 3\sqrt[3]{2\cdot x}$$

Cube करने पर

$x^6 = 27 . 2 . x$

$x^5 = 54$

$\boxed{x = (54)^{1/5}}$ Ans

Trick

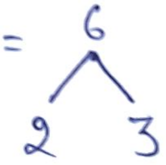
$a \times b = \text{No.}$ If Symbol (+) → Big No.
$a - b = 1$ (−) → Small No.

$\boxed{\sqrt{x + \sqrt{x + \sqrt{x \ldots\ldots \infty}}} \Rightarrow \frac{1 + \sqrt{4x + 1}}{2}}$

Note :→ इस Trick में दिये हुए प्रश्न में जो संख्या दी है उसके factor करते हैं factor का गुणा अंतर 1 होना चाहिए और यदि प्रश्न (+) में है तो factor की बड़ी संख्या उत्तर होगी और यदि प्रश्न (−) में है तो factor की छोटी संख्या उत्तर होगी।

10 $\sqrt{6 + \sqrt{6 + \sqrt{6 \ldots\ldots \infty}}}$

Solve (I) $\sqrt{6 + \sqrt{6 + \sqrt{6 \ldots \infty}}} =$



Ans ⇒ ③

(II) $\sqrt{6 + \sqrt{6 + \sqrt{6 \ldots \infty}}} = \frac{1 + \sqrt{4 \times 6 + 1}}{2}$

$= \frac{1 + \sqrt{25}}{2} = \frac{1 + 5}{2}$

$= \frac{6}{2} = 3$ Ans

11 $\sqrt{6-\sqrt{6-\sqrt{6\ldots\infty}}}$

Solve

$\sqrt{6-\sqrt{6-\sqrt{6\ldots\infty}}} = \begin{array}{c} 6 \\ \wedge \\ 2 \quad 3 \end{array}$ Ans $\Rightarrow$ 2

12 $\sqrt{72+\sqrt{72\ldots\ldots\infty}}$

Solve

$\sqrt{72+\sqrt{72\ldots\infty}} = \begin{array}{c} 72 \\ \wedge \\ 8 \quad 9 \end{array}$ Ans $\Rightarrow$ 9

Imp

13 $\sqrt{1+\sqrt{1+\sqrt{1\ldots\infty}}}$

Solve

$$\sqrt{1+\sqrt{1+\sqrt{1\ldots\infty}}} = \frac{1+\sqrt{4\times1+1}}{2} = \frac{1+\sqrt{5}}{2} = \frac{1+2.25}{2}$$

$$= \frac{3.25}{2}$$

$= 1.6$ Ans

⑭ If $x = \sqrt{20+\sqrt{20+\sqrt{20\ldots\infty}}}$

$y = \sqrt{6-\sqrt{6-\sqrt{6\ldots\infty}}}$

Solve

$x = \begin{array}{c} 20 \\ \wedge \\ 4 \quad 5 \end{array} \Rightarrow 5$ $y = \begin{array}{c} 6 \\ \wedge \\ 2 \quad 3 \end{array} = 2$

(i) $x+y \Rightarrow 5+2 = 7$ (ii) $x \times y = 5\times2 = 10$

(iii) $\frac{x}{y} = \frac{5}{2}$ (iv) $x-y = 5-2 = 3$

15⇒ $x = \sqrt{7+\sqrt{7+\sqrt{7 \ldots}}} \infty$

$y = \sqrt{7-\sqrt{7-\sqrt{7 \ldots}}} \infty$

Solve → $x - y = 1$

[Note → इस प्रकार के प्रश्नों में $x-y$ हमेशा 1 ही होगा।]

16⇒ $\sqrt{3\sqrt{3}}$

Solve ⇒ $\sqrt{3\sqrt{3}} \Rightarrow 3^{1/2}\, 3^{1/4} \Rightarrow 3^{1/2+1/4} \Rightarrow 3^{\frac{2+1}{4}}$

$\Rightarrow 3^{3/4}$ Ans

[**Note** ⇒ इस प्रकार के प्रश्नों में जितने $\sqrt{\sqrt{}}$ होते है $\sqrt{} = 2$ का आपस में गुणा करके $\frac{}{x}$ लिखते है और $\frac{x-1}{x}$ लिखते हैं।]

17⇒ $\sqrt{5\sqrt{5\sqrt{5}}}$

Solve ⇒ $\sqrt{5\sqrt{5\sqrt{5}}} = 5^{\frac{7}{8}}$ Ans

18⇒ $\sqrt{2\sqrt{4\sqrt{8}}}$

Solve $\sqrt{2\sqrt{4\sqrt{8}}} \Rightarrow 2^{1/2} . 4^{1/4} . 8^{1/8}$

$\Rightarrow 2^{1/2} . 2^{1/2} . 2^{3/8}$

$\Rightarrow 2^{1/2+1/2+3/8} \Rightarrow 2^{\frac{4+4+3}{8}}$

$\Rightarrow 2^{11/8}$

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19 $\sqrt{7\sqrt{7\sqrt{7 \cdots}}}\ \infty = (343)^{x-1}$

Solve

$$\sqrt{7\sqrt{7\sqrt{7 \cdots}}}\ \infty = (343)^{x-1}$$

$$7 = (7)^{3x-3}$$

$$3x \div 3 = 1$$

$$3x = 4$$

$$\boxed{x = \frac{4}{3}}$$ Ans

20 $\sqrt{7\sqrt{7\sqrt{7 \cdots}}}\ \infty = (343)^{2x+1}$

Solve

$$\sqrt{7\sqrt{7\sqrt{7 \cdots}}}\ \infty = (343)^{2x-1}$$

$$7 = (7)^{6x-3}$$

$$6x - 3 = 1$$

$$6x = 4$$

$$\boxed{x = \frac{2}{3}}$$ Ans

㉑ $\sqrt{7\sqrt{7\sqrt{7}}} = (343)^{2x-1}$

Solve

$$\sqrt{7\sqrt{7\sqrt{7}}} = (343)^{2x-1}$$

$$7^{\frac{7}{8}} = (7)^{6x-3}$$

$$\frac{7}{8} = 6x - 3$$

$$6x = \frac{7}{8} + 3$$

$$x = \frac{31}{8} * 6$$

$$x = \frac{31}{48}$$

Note :→ इस प्रकार के प्रश्नों में दी हुई अंतिम संख्या बड़ी को दी हुई सबसे छोटी संख्या में घटाते हैं।

22 $$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \text{-------} + \frac{1}{\sqrt{99}+\sqrt{100}}$$

Solve

$$\sqrt{100} - \cancel{\sqrt{\ }}1 \Rightarrow 10 - 1 = 9 \text{ Ans}$$

Note :→ ① Differen Check करते हैं जितना निचे है उतना ही ऊपर हो

② Pattern Same

23 $$\frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \text{-------} + \frac{1}{\sqrt{168}-\sqrt{169}}$$

Solve

$$\sqrt{169} - \sqrt{4} = 13 - 2 = 11 \text{ A}$$

24 $$\frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}} + \text{-------} + \frac{1}{\sqrt{22}+\sqrt{23}}$$

Solve

$$\sqrt{23} - \sqrt{7}$$

Ans

25 $$\frac{2}{\sqrt{1}+\sqrt{3}} + \frac{2}{\sqrt{3}+\sqrt{5}} + \frac{2}{\sqrt{5}+\sqrt{7}} + \text{-------} + \frac{2}{\sqrt{47}+\sqrt{49}}$$

Solve

$$\sqrt{49} - \sqrt{1} = 7 - 1 = 6 \text{ A}$$

Find the Gratest & Smallest

26. $(\sqrt{3}+\sqrt{5})$, $(\sqrt{5}+\sqrt{7})$, $(\sqrt{7}+\sqrt{9})$, $(\sqrt{9}+\sqrt{11})$

Note :→ इस प्रकार के प्रश्नों में यदि ⊕ में है तो जो सबसे बड़ी संख्या होती है वह Gratest और जो सबसे छोटी होती है वह Smallest होती है।

यदि ⊖ में है तो जो सबसे छोटी होती है वह Gratest और जो सबसे बड़ी होती है वह Smallest होती है।

★ इसमें Defference Check करते हैं।

Solve Gratest ⇒ $\sqrt{9}+\sqrt{11}$, Smallest ⇒ $\sqrt{3}+\sqrt{5}$

27. $(\sqrt{5}-\sqrt{3})$, $(\sqrt{7}-\sqrt{5})$, $(\sqrt{9}-\sqrt{7})$, $(\sqrt{11}-\sqrt{9})$

Solve Gratest ⇒ $(\sqrt{5}-\sqrt{3})$, Smallest ⇒ $(\sqrt{11}-\sqrt{9})$

Advance Level Question →*←

① If $x = 2-\sqrt{3}$, $xy = 1$ find $x^2+\frac{1}{x^2}$

Solve $x = \frac{1}{y}$, $y = \frac{1}{x}$

Note → Diff. of Sq. = 1
Symbal of Reverse

$\left[\frac{1}{x} = 2+\sqrt{3}\right]$

$$x = 2-\cancel{\sqrt{3}}$$
$$\frac{1}{x} = 2+\cancel{\sqrt{3}}$$
$$x+\frac{1}{x} = 4$$

$$\boxed{x^2+\frac{1}{x^2} = 14}$$

Ans

② If $x = \sqrt{3} + \sqrt{2}$, $xy = 1$ find ⇒ $x^2 + \frac{1}{x^2}$

Solve

$$x = \sqrt{3} + \cancel{\sqrt{2}}$$

$$\frac{1}{x} = \sqrt{3} - \cancel{\sqrt{2}}$$

$$x + \frac{1}{x} = 2\sqrt{3}$$

$$\boxed{x^2 + \frac{1}{x^2} = 10}$$ Ans

③ If $x = 2 - \sqrt{3}$, $xy = 1$ find ⇒ $x^3 + \frac{1}{x^3}$

Solve

$$x = 2 - \cancel{\sqrt{3}}$$

$$\frac{1}{x} = 2 + \cancel{\sqrt{3}}$$

$$x + \frac{1}{x} = 4$$

$$\boxed{x^3 + \frac{1}{x^3} = 52}$$ Ans

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④ If $x = \sqrt{3} + \sqrt{2}$
$y = \sqrt{3} - \sqrt{2}$
find $\frac{1}{1+x} + \frac{1}{1+y}$

Solve

$$x = \sqrt{3} + \sqrt{2}$$
$$y = \sqrt{3} - \sqrt{2}$$

$$x = \frac{1}{y}, \quad y = \frac{1}{x}$$

$$\frac{1}{1+x} + \frac{1}{1+y} \Rightarrow \frac{1}{1+x} + \frac{1}{1+\frac{1}{x}} \Rightarrow \frac{1}{1+x} + \frac{x}{x+1}$$

$$\Rightarrow \frac{\cancel{1+x}}{\cancel{x+1}} = 1$$

[Note:→ इस प्रकार के सभी प्रश्नों का उत्तर हमेशा 1 होता है।]

⑤ If $x = 2 + \sqrt{3}$
$y = 2 - \sqrt{3}$
find $\frac{1}{1+x} + \frac{1}{1+y}$

Solve Ans ⇒ 1

⑥ If $x = \frac{1}{2+\sqrt{3}}$, $y = \frac{1}{2-\sqrt{3}}$ find $(1+x)^{-1} + (1+y)^{-1}$

Solve

Ans ⇒ 1

⑦ $x = (\sqrt{3}+\sqrt{2})^{-3}$ find $(1+x)^{-1} + (1+y)^{-1}$

$y = (\sqrt{3}-\sqrt{2})^{-2}$

Solve

Ans → 1 A

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⑧ If $x = 2+\sqrt{3}$ find $\frac{x^3+y^3}{x^2+y^2}$

$y = 2-\sqrt{3}$

Solve

$y = \frac{1}{x}$

$x = 2+\sqrt{3}$

$\frac{1}{x} = 2-\sqrt{3}$

$x + \frac{1}{x} = 4$

$$\Rightarrow \frac{x^3 + \frac{1}{x^3}}{x^2 + \frac{1}{x^2}}$$

$$\Rightarrow \frac{\cancel{52}}{\cancel{14}} \Rightarrow \frac{26}{7}$$ Ans

⑨ If $x = 3+2\sqrt{2}$ find $\frac{x^3+y^3+3xy}{x^2+y^2-5xy}$

$xy = 1$

Solve

$y = \frac{1}{x} \Rightarrow x + \frac{1}{x} = 6$

$$\frac{x^3 + \frac{1}{x^3} + 3}{x^2 + \frac{1}{x^2} - 5} \Rightarrow \frac{198+3}{34-5} = \frac{201}{29}$$ Ans

10 If $\sqrt{3x^2-12x+19}+\sqrt{3x^2-12x-11}=6$

find $\sqrt{3x^2-12x+19}-\sqrt{3x^2-12x-11}=?$

Solve

$$\sqrt{3x^2-12x+19}+\sqrt{3x^2-12x-11}=6$$

$$\sqrt{3x^2-12x+19}-\sqrt{3x^2-12x-11}=t$$

दोनों का आपस में गुणा करने पर

$$\cancel{3x^2}-\cancel{12x}+19-\cancel{3x^2}+\cancel{12x}+11=6t$$

$$30=6t$$

$$\boxed{t=5}$$

Trick
Note ⇒ इस प्रकार के प्रश्नों में

$$\sqrt{3x^2-12x+19}+\sqrt{3x^2-12x-11}=6$$

$$\frac{\text{add}}{\text{devide}}=?$$

11 If $\sqrt{x^2-12x+7}+\sqrt{x^2-12x+7}=2$

find $\sqrt{x^2-12x+7}-\sqrt{x^2-12x+7}=?$

Solve

$$\frac{7+7}{2}=\frac{14}{2}=7$$

12

$2^{2^2} \times 2^{2^{-2}}$

Solve

$$2^{2^2} \times 2^{2^{-2}} \Rightarrow 2^4 \times 2^{\frac{1}{2^2}} \Rightarrow 2^4 \times 2^{1/4}$$

$$\Rightarrow 2^{4+\frac{1}{4}} \Rightarrow 2^{17/4} \text{ Ans}$$

13

$2^{2^3} \times 2^{2^{-3}}$

Solve $2^8 \times 2^{1/8} \Rightarrow 2^{8+\frac{1}{8}} \Rightarrow 2^{65/8}$ Ans

14

$2^{2^2} \times 2^{2^{-3}}$

Solve $2^4 \times 2^{1/8} \Rightarrow 2^{4+\frac{1}{8}} \Rightarrow 2^{33/8}$ Ans

15 $2^{2x-1} = 8^3$ find = $x = ?$

Solve

$$2^{2x-1} = 8^3 \quad (2^3)^3$$

$$2^{2x-1} = 2^9 \Rightarrow 2x-1 = 9 \Rightarrow 2x = 10 \Rightarrow \boxed{x=5}$$

16 $27^{3x+1} = 81^6$ find $x = ?$

Solve

$$27^{3x+1} = 81^6 \Rightarrow 3^{\cancel{3}(3x+1)} = 3^{\overset{2}{\cancel{6}} \times 4} \Rightarrow 3x+1 = 8$$

$$\Rightarrow 3x = 7$$

$$\Rightarrow \boxed{x = \frac{7}{3}}$$

17 $2^{x-1}+2^{x+1}=320$

Solve

$$2^{x+1}+2^{x+1}=320$$

$$2^{x}\cdot 2^{-1}+2^{x}\cdot 2=320$$

$$2^{x}\left(\frac{1}{2}+2\right)=320$$

$$2^{x}\times\frac{5}{2}=320 \Rightarrow 2^{x}=64\times 2=128$$

$$\Rightarrow 2^{x}=2^{7}$$

$$\Rightarrow \boxed{x=7}$$

18 $2^{x-1}+2^{x+1}=640$

Solve

$$2^{x-1}+2^{x+1}=640$$

$$2^{x}\cdot 2^{-1}+2^{x}\cdot 2=640$$

$$2^{x}\times\frac{5}{2}=640$$

$$2^{x}=128\times 2 \Rightarrow 2^{x}=256$$

$$\Rightarrow 2^{x}=2^{8}$$

$$\Rightarrow \boxed{x=8}$$

Ratio & Proportion

$a:b :: c:d$

$\frac{a}{b} = \frac{c}{d}$

$\frac{a+b}{a-b} = \frac{c+d}{c-d}$

$:$ ⇒ भाग (÷)

$::$ ⇒ बराबर (=)

① If $x = \frac{4ab}{a+b}$, find $\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$

Solve

Solve

$$x = \frac{4ab}{a+b}$$

$\frac{x}{2b} = \frac{2a}{a+b}$

$\Rightarrow \frac{2a+a+b}{2a-a-b}$

$\frac{x+2b}{x-2b} \Rightarrow \frac{3a+b}{a-b}$

$\frac{x}{2a} = \frac{2b}{a+b}$

$\frac{x+2a}{x-2b} \Rightarrow \frac{2b+a+b}{2b-a-b}$

$\frac{x+2a}{x-2b} \Rightarrow \frac{3b+a}{b-a}$

$\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} \Rightarrow \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$

$\Rightarrow \frac{3b+a}{b-a} - \frac{3a+b}{b-a}$

$\Rightarrow \frac{3b+a-3a-b}{b-a} = \frac{2b-2a}{b-a}$

$= 2\left(\cancel{\frac{b-a}{b-a}}\right)$

$\boxed{\text{Ans} \Rightarrow 2}$

[**Note** :→ इस प्रकार के सभी प्रश्नों का उत्तर हमेशा 2 होता है।]

2 If $x = \frac{2ab}{a+b}$ find $\frac{x+a}{x-a} + \frac{x+b}{x-b}$

Solve

Ans ⇒ 2

3 If $x = \frac{6ab}{a+b}$ find $\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b}$

Solve

Ans → 2

4 If $x = \frac{8ab}{a+b}$ find $\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b}$

Solve

Ans ⇒ 2

5 If $x = \frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$ find $\frac{x+\sqrt{20}}{x-\sqrt{20}} + \frac{x+\sqrt{12}}{x-\sqrt{12}}$

Solve

Ans ⇒ 2

17/12/16

(1) $\sqrt{4+2\sqrt{3}}$

Solve

$$\sqrt{4+\underbrace{2\sqrt{3}}_{2\sqrt{a}\sqrt{b}}}$$

$$2\sqrt{3}\sqrt{1}$$

$$\sqrt{(\sqrt{3}+\sqrt{1})^2} \Rightarrow \sqrt{3}+1$$

Ans

Note :→ इस प्रकार के प्रश्नों में $[a^2+b^2+2a.b]$ से Solve करते हैं।

2 $\sqrt{6+2\sqrt{5}}$

Solve

$(\sqrt{5}+1)$ Ans

3 $\sqrt{8+2\sqrt{15}}$

Solve

$(\sqrt{5}+\sqrt{3})$ Ans

4 $\sqrt{10+2\sqrt{21}}$

Solve

$(\sqrt{7}+\sqrt{3})$ Ans

5 $\sqrt{19+8\sqrt{3}}$

Solve

$\sqrt{19+8\sqrt{3}}$

$2 \cdot a \cdot b$

$2 \cdot 4 \cdot \sqrt{3}$

$2 \cdot \sqrt{16} \cdot \sqrt{3}$

$\Rightarrow \sqrt{(\sqrt{16}+\sqrt{3})^2} \Rightarrow (\sqrt{16}+\sqrt{3})$ Ans

6. $\sqrt{7+4\sqrt{3}}$

Solve

$\sqrt{7+4\sqrt{3}}$

$2 \cdot a \cdot b$

$2 \cdot 2 \cdot \sqrt{3}$

$2 \cdot \sqrt{4} \cdot \sqrt{3}$

$\Rightarrow \sqrt{(\sqrt{4}+\sqrt{3})^2} \Rightarrow (2+\sqrt{3})$ Ans

Imp

1. If $x = \frac{\sqrt{3}}{2}$ find $\sqrt{1+x}$

Solve

$$\sqrt{1+x} \Rightarrow \sqrt{1+\frac{\sqrt{3}}{2}} \Rightarrow \sqrt{\frac{(2+\sqrt{3})\times 2}{(2)\times 2}}$$

$$\Rightarrow \sqrt{\frac{4+2\sqrt{3}}{4}}$$

$$\Rightarrow \sqrt{\frac{(\sqrt{3}+1)^2}{4}} \Rightarrow \boxed{\frac{\sqrt{3}+1}{2}}$$

$$\boxed{\sqrt{1+x} = \frac{\sqrt{3}+1}{2}}$$

2. If $x = \frac{\sqrt{3}}{2}$ find $\sqrt{1-x}$

Solve

$$\sqrt{1-x} \Rightarrow \sqrt{1-\frac{\sqrt{3}}{2}} \Rightarrow \sqrt{\frac{(2-\sqrt{3})\times 2}{(2)\times 2}}$$

$$\Rightarrow \sqrt{\frac{4-2\sqrt{3}}{4}}$$

$$\Rightarrow \sqrt{\frac{(\sqrt{3}-1)^2}{4}} \Rightarrow \boxed{\frac{\sqrt{3}-1}{2}}$$

$$\boxed{\sqrt{1-x} = \frac{\sqrt{3}-1}{2}}$$

3. If $x = \frac{\sqrt{3}}{2}$ find $\sqrt{1+x} + \sqrt{1-x}$

Solve

$$\sqrt{1+x} + \sqrt{1-x} = \frac{\sqrt{3}+1}{2} + \frac{\sqrt{3}-1}{2}$$

$$= \frac{\sqrt{3}\cancel{+1}+\sqrt{3}\cancel{-1}}{2}$$

$$= \frac{\cancel{2}\sqrt{3}}{\cancel{2}} = \sqrt{3} \,//$$

4. If $x = \frac{\sqrt{3}}{2}$ find $\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}$

Solve

$$\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}} \Rightarrow \frac{\frac{\sqrt{3}+1}{2} + \frac{\sqrt{3}-1}{2}}{\frac{\sqrt{3}+1}{2} + \frac{\sqrt{3}-1}{2}}$$

$$\Rightarrow \frac{\cancel{2}\sqrt{3}}{\cancel{2}} = \sqrt{3} \,//$$

5. If $x = \frac{\sqrt{3}}{2}$ find $\frac{\sqrt{1+x}}{1+\sqrt{1+x}} + \frac{\sqrt{1-x}}{1-\sqrt{1-x}}$

Solve

$$\frac{\sqrt{1+x}}{1+\sqrt{1+x}} + \frac{\sqrt{1-x}}{1-\sqrt{1-x}} \Rightarrow \frac{\frac{\sqrt{3}+1}{2}}{1+\frac{\sqrt{3}+1}{2}} + \frac{\frac{\sqrt{3}-1}{2}}{1-\frac{\sqrt{3}-1}{2}}$$

$$\Rightarrow \frac{\frac{\sqrt{3}+1}{2}}{\frac{2+\sqrt{3}+1}{2}} + \frac{\frac{\sqrt{3}-1}{2}}{\frac{2-\sqrt{3}+1}{2}}$$

$$\Rightarrow \frac{\frac{\sqrt{3}+1}{2}}{\frac{3+\sqrt{3}}{2}} + \frac{\frac{\sqrt{3}-1}{2}}{\frac{3-\sqrt{3}}{2}}$$

$$\Rightarrow \frac{\cancel{\sqrt{3}+1}}{\sqrt{3}(\cancel{\sqrt{3}+1})} + \frac{\cancel{\sqrt{3}-1}}{\sqrt{3}(\cancel{\sqrt{3}-1})}$$

$$\Rightarrow \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} \text{ Ans}$$

6 If $x = \frac{2\sqrt{10}}{7}$ find $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$

Solve

$$\sqrt{1+x} = \sqrt{1+\frac{2\sqrt{10}}{7}} \Rightarrow \sqrt{\frac{7+2\sqrt{10}}{7}}$$

$$\Rightarrow \frac{\sqrt{5}+\sqrt{2}}{\sqrt{7}}$$

$$\sqrt{1-x} \Rightarrow \frac{\sqrt{5}-\sqrt{2}}{\sqrt{7}}$$

$$\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} \Rightarrow \frac{\frac{\sqrt{5}+\sqrt{2}}{\sqrt{7}}+\frac{\sqrt{5}-\sqrt{2}}{\sqrt{7}}}{\frac{\sqrt{5}+\sqrt{2}}{\sqrt{7}}-\frac{\sqrt{5}-\sqrt{2}}{\sqrt{7}}}$$

$$\Rightarrow \frac{\sqrt{5}+\cancel{\sqrt{2}}+\sqrt{5}-\cancel{\sqrt{2}}}{\cancel{\sqrt{5}}+\sqrt{2}-\cancel{\sqrt{5}}+\sqrt{2}}$$

$$\Rightarrow \cancel{\frac{\cancel{2}\sqrt{5}}{\cancel{2}\sqrt{3}}} \quad \frac{\cancel{2}\sqrt{5}}{\cancel{2}\sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{5}}{\sqrt{2}}$$

rn Series :→

Note →: ① Pattern
② Difference

1 $\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}$

Solve

Concept

$\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}$

2 3 3 4 4 5 5 6 6 7

$\frac{1}{2}+\cancel{\frac{1}{3}}+\cancel{\frac{1}{3}}-\cancel{\frac{1}{4}}+\cancel{\frac{1}{4}}-\cancel{\frac{1}{5}}+\cancel{\frac{1}{5}}-\cancel{\frac{1}{6}}+\cancel{\frac{1}{6}}-\frac{1}{7}$

$\frac{1}{2}-\frac{1}{7}=\frac{5}{14}$ **Ans**

Trick

$\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}$

② 3 3 4 4 5 5 6 6 ⑦

$\frac{1}{2}-\frac{1}{7}\Rightarrow\frac{5}{14}$ //

② $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}$

Solve

$\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}$

① 2 2 3 6 7 7 ⑧

$$\frac{1}{1}-\frac{1}{8}=\frac{7}{8}$$ Ans

3

$$\frac{2}{1.3}+\frac{2}{3.5}+\frac{2}{5.7}+\frac{2}{7.9}+\frac{2}{9.11}$$

Solve

$$\frac{2}{1.3}+\frac{2}{3.5}+\frac{2}{5.7}+\frac{2}{7.9}+\frac{2}{9.11}$$

$$\frac{1}{1}-\frac{1}{11}\Rightarrow\frac{10}{11}$$ Ans

4

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}$$

Solve

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}$$

$$\Rightarrow\frac{2\times}{2\times}\left[\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}\right]$$

$$\Rightarrow\frac{1}{2}\left[\frac{2}{1.3}+\frac{2}{3.5}+\frac{2}{5.7}+\frac{2}{7.9}\right]$$

$$\Rightarrow\frac{1}{2}\left[\frac{1}{1}-\frac{1}{9}\right]$$

$$\Rightarrow\frac{1}{\cancel{2}}\times\frac{\cancel{8}^{\,4}}{9}\Rightarrow 4/9$$ Ans

5

$$\frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}+------+\frac{19}{9^2.10^2}$$

Solve

$$\frac{3}{1.4}+\frac{5}{4.9}+\frac{7}{9.16}------+\frac{19}{81.100}$$

$$\frac{1}{1}-\frac{1}{100}=\frac{99}{100}$$ //

In Multiplication →

★① $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\cdots\left(1+\frac{1}{n}\right)$

Solve

$$\frac{\cancel{3}}{2}\times\frac{\cancel{4}}{\cancel{3}}\times\frac{\cancel{5}}{\cancel{4}}\times\cancel{\cdots}\cdots\times\left(\frac{n+1}{\cancel{n}}\right)$$

$$\boxed{\left[\frac{n+1}{2}\right]}$$

Q $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\cdots\left(1+\frac{1}{19}\right)$

Solve

$$\frac{\cancel{3}}{2}\times\frac{\cancel{4}}{\cancel{3}}\times\frac{\cancel{5}}{\cancel{4}}\times\cancel{\cdots}\cdots\times\frac{20}{\cancel{19}}$$

$$\frac{\cancel{20}}{\cancel{2}} = 10$$

★ $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\cdots\left(1-\frac{1}{n}\right)$

Solve

$$\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\cancel{\cdots}\cdots\times\left(\frac{\cancel{n-1}}{n}\right)$$

$$\boxed{\left[\frac{1}{n}\right]}$$

Q $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\cdots\left(1-\frac{1}{20}\right)$

Solve

$$\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\cancel{\cdots}\cdots\times\frac{\cancel{19}}{20}$$

$$\frac{1}{20}$$ Ans

1. $2^x = 3^y = 6^{-z}$ find $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = ?$

Solve

$$2^x = 3^y = 6^{-z} = K \text{ (let)}$$

$2^x = K$	$3^y = K$	$6^{-z} = K$
$2 = K^{1/x}$	$3 = K^{1/y}$	$6 = K^{-1/z}$

$2 \times 3 = 6$

$\downarrow \quad \searrow$

$$\boxed{K^{1/x} \times K^{1/y}} = K^{-1/z}$$

$$K^{1/x + \frac{1}{y}} = K^{-1/z}$$

$$\frac{1}{x}+\frac{1}{y} = -\frac{1}{z}$$

$$\boxed{\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0}$$

② $3^x = 4^y = 12^z$ find $y = ?$

Solve ⇒

$$3^x = 4^y = 12^z = K \text{ (let)}$$

$3^x = K$	$4^y = K$	$12^z = K$
$3 = K^{1/x}$	$4 = K^{1/y}$	$12 = K^{1/z}$

$3 \times 4 = 12$

$\downarrow \quad \downarrow \quad \downarrow$

$$K^{\frac{1}{x}} \times K^{\frac{1}{y}} = K^{1/z}$$

$$K^{1/x + \frac{1}{y}} = K^{1/z}$$

$$\frac{1}{x}+\frac{1}{y} = \frac{1}{z}$$

$$\frac{1}{y} = \frac{1}{z} - \frac{1}{x}$$

$$\frac{1}{y} = \frac{x-z}{zx}$$

$$\boxed{y = \frac{zx}{x-z}}$$

3. $x^a = y^b = z^c$ & $y^2 = zx$ find $\frac{1}{a} + \frac{1}{c} = ?$

Solve

$$x^a = y^b = z^c = K \text{ (let)}$$

$x = K^{1/a}$

$y = K^{1/b}$

$z = K^{1/c}$

$y^2 = zx$

$K^{2/b} = K^{1/c} \times K^{1/a}$

$K^{2/b} = K^{1/c + \frac{1}{a}}$

$$\boxed{\frac{2}{b} = \frac{1}{c} + \frac{1}{a}}$$

4. $a^{1/m} = b^{1/n} = c^{1/p}$ & $abc = 1$ find $m+n+p = ?$

Solve

$$a^{1/m} = b^{1/n} = c^{1/p} = K \text{ (let)}$$

$a = K^m$

$b = K^n$

$c = K^p$

$abc = 1$

$K^m \times K^n \times K^p = 1$

$K^{m+n+p} = 1$ or K^0

$$\boxed{m+n+p = 0}$$

Next Type :→

① $\left(\frac{x^a}{x^b}\right)^{(a+b)} \times \left(\frac{x^b}{x^c}\right)^{(b+c)} \times \left(\frac{x^c}{x^a}\right)^{(c+a)}$

Solve $x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$

$\Rightarrow x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$

$\Rightarrow x^{\cancel{a^2} - \cancel{b^2} + \cancel{b^2} - \cancel{c^2} + \cancel{c^2} - \cancel{a^2}} = \boxed{x^0 = 1}$

Note :→ इस प्रकार के सभी प्रश्नों का उत्तर 1 ही होता है।

2 $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$

Solve

$$\frac{1}{1+\frac{x^a}{x^b}} + \frac{1}{1+\frac{x^b}{x^a}}$$

$$\frac{x^b}{x^b+x^a} + \frac{x^a}{x^a+x^b} \Rightarrow \frac{\cancel{x^b+x^a}}{\cancel{x^a+x^b}} = 1$$

③ $[x^{(a+b)}]^{(a-b)} \times [x^{(b+c)}]^{(b-c)} \times [x^{(c+a)}]^{(c-a)}$

Ans ⇒ 1

④ $\frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{c-a}+x^{b-a}} + \frac{1}{1+x^{a-c}+x^{b-c}}$

Ans → 1

Next Type :→

1 If $x+\frac{1}{x}=1$, $x^3=?$

Solve

$$x+\frac{1}{x}=1 \Rightarrow \frac{x^2+1}{x}=1$$

$a = x$, $b = 1$

$$\Rightarrow x^2+1=x$$

$$\Rightarrow x^2-x+1=0$$

$$x^3+1^3=(x+1)(x^2+1-x)$$

$$x^3+1^3=(x+1)\,0$$

$$\boxed{x^3+1^3=0} \qquad \boxed{x^3=-1}$$

2. $x+\frac{1}{x} = -1 \qquad x^3 = ?$

Solve

$$x+\frac{1}{x} = -1$$

$$\frac{x^2+1}{x} = -1 \Rightarrow x^2+1 = -x$$

$$x^2+x+1=0$$

$$x^3 - 1^3 = (x-1)(x^2+x+1).$$

$$x^3-1^3 = (x-1)\,0$$

$$x^3-1^3 = 0$$

$$\boxed{x^3 = 1}$$

Trick

$$\frac{x}{1}+\frac{1}{x} = \underbrace{+1}_{} \qquad \boxed{x^3 = -1}$$

Sing. (+ → -)

$(1)^3$

$$\frac{x}{1}+\frac{1}{x} = \underbrace{-1}_{} \qquad \boxed{x^3 = 1}$$

Sing (- → +)

$(1)^3$

Note :→

$x+\frac{1}{x} = -1, \quad x^3 = 1$

$x+\frac{1}{x} = 1, \quad x^3 = -1$

3. If $\frac{x}{3}+\frac{3}{x} = 1 \qquad x^3 = ?$

Solve

$$x^3 = -27$$

4. If $\frac{x}{4}+\frac{4}{x} = -1 \qquad x^3 = ?$

Solve

$$x^3 = 64$$

Q If $x+\frac{1}{x}=1$

find

(i) $x^9+x^6+x^3+1$

Solve

$$(x^3)^3+(x^3)^2+x^3+1$$

$$\cancel{-1}\cancel{+1}\cancel{-1}\cancel{+1} \Rightarrow 0$$

$x+\frac{1}{x}=1$ $\boxed{x^3=-1}$

$x^3+1=0$

$\boxed{x^3 \overset{}{\leftrightarrow} + x^0 = 0}$

(ii) $x^{99}+x^{96}+x^{90}+x^{87}$

Solve

$$\overbrace{x^{99}+x^{96}}^{3}+\overbrace{\cancel{x}^{90}+\cancel{x}^{87}}^{3}$$

$$0+0=0$$

(iii) $x^{60}+\frac{1}{x^{60}}$

Solve

$$(x^3)^{20}+\frac{1}{(x^3)^{20}} \Rightarrow 1+1=2$$

(iv) $x^{61}+\frac{1}{x^{61}}$

Solve

$$x\cdot(x^3)^{20}+\frac{1}{x\cdot(x^3)^{20}} \Rightarrow \boxed{x+\frac{1}{x} \Rightarrow 1}.$$

1 If $\frac{a}{b}+\frac{b}{a}=1$ find a^3+b^3

Solve

$a^3+b^3=0$

$a^3=-b^3$

$a^3+b^3=\cancel{-b^3}\cancel{+b^3}$

$\boxed{a^3+b^3=0}$

2 If $\frac{a}{b}+\frac{b}{a}=-1$ find a^3-b^3

Solve

$a^3=b^3$

$a^3-b^3=\cancel{b^3}\cancel{-b^3}$

$=0$

Note :→ इस प्रकार के प्रश्नों के उत्तर 0 होता है।

3 If $a=\frac{b^2}{b-a}$ find a^3+b^3

Solve

$ab-a^2=b^2$

$a^2+b^2-ab=0$

$a^3+b^3=(a+b)(a^2+b^2-ab)$

$=(a+b)\,0$

$\boxed{a^3+b^3=0}$

Next Type :→

1 If $x^2-5x+1=0$ find $x+\frac{1}{x}=?$

Solve $x^2-5x+1=0$

divide x

$\frac{x^{\cancel{2}}}{\cancel{x}}-\frac{5\cancel{x}}{\cancel{x}}+\frac{1}{x}=0$

$\boxed{x+\frac{1}{x}=5}$

2 If $x^2-6x+1=6$ find $x^2+\frac{1}{x^2}=?$

Solve

$$x^2-6x+1=0$$

$$x+\frac{1}{x}=6 \Rightarrow \boxed{x^2+\frac{1}{x^2}=34}$$

3 If $x^2-4x+1=0$ find $x^3+\frac{1}{x^3}=?$

Solve

$$x^2-4x+1=0$$

$$x+\frac{1}{x}=4 \Rightarrow \boxed{x^3+\frac{1}{x^3}=52}$$

4 $3x-\frac{3}{x}=6$ find $x^2+\frac{1}{x^2}=?$

$$3\left(x-\frac{1}{x}\right)=6$$

$$x-\frac{1}{x}=2 \qquad \boxed{x^2+\frac{1}{x^2}=6}$$

5 $\frac{3p}{p^2-3p+3}=\frac{1}{6}$ find $p+\frac{3}{p}=?$

Solve

$$\frac{p^2-3p+3}{3p}=6 \Rightarrow p+\frac{3}{p}=18+3$$

$$\boxed{p+\frac{3}{p}=\cancel{9}21}$$

6 $\frac{4p}{p^2-4p+4}=\frac{1}{4}$ find $p^2+\frac{16}{p^2}=?$

Solve

$$\frac{p^2-4p+4}{4p}=4 \implies \boxed{p+\frac{4}{p}=20}$$

Sq. करने पर

$$p^2+\frac{16}{p^2}+2\cdot\cancel{p}\cdot\frac{4}{\cancel{p}}=400$$

$$\boxed{p^2+\frac{16}{p^2}=392}$$

7 $4x-\frac{2}{3x}=4$ find $8x^3-\frac{1}{27x^3}=?$

Solve

divide by 2

$$2x-\frac{1}{3x}=2$$

Cube करने पर

$$8x^3-\frac{1}{27x^3}-\cancel{3}\times 2\cancel{x}\times\frac{1}{\cancel{3x}}\left(2x-\frac{1}{3x}\right)=8$$

$$\boxed{8x^3-\frac{1}{27x^3}=12}$$

8 $2x-\frac{1}{2x}=6$ find $x^2+\frac{1}{16x^2}=?$

Solve

divide by 2

$$x-\frac{1}{4x}=3$$

Sq. करने पर

$$x^2+\frac{1}{16x^2}-\cancel{2}\times\cancel{x}\times\frac{1}{\underset{2}{\cancel{4x}}}=9$$

$$x^2+\frac{1}{16x^2}=9+\frac{1}{2}$$

$$\boxed{x^2+\frac{1}{16x^2}=9\frac{1}{2}}$$

Next Type :→

①

$$a = \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$$

$$b = \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$

find $\dfrac{a^2+b^2+ab}{a^2+b^2-ab} = ?$

Solve

$\boxed{a = \frac{1}{b}}$ $\boxed{b = \frac{1}{a}}$

$$\frac{a^2+\frac{1}{a^2}+1}{a^2+\frac{1}{a^2}-1} = ?$$

$a = \dfrac{12 - 2\sqrt{35}}{2} \Rightarrow 6-\cancel{\sqrt{35}}$

$\dfrac{1}{a}$ or $b = \dfrac{12 + 2\sqrt{35}}{2} \Rightarrow 6+\cancel{\sqrt{35}}$

$a+\dfrac{1}{a} = 12$

$$\boxed{\cancel{\frac{574+1}{574-1}} \Rightarrow \boxed{\cancel{\frac{575}{573}}}}$$

$$\boxed{\frac{145}{141}}$$

(2) $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ find ⇒ $\frac{a^2+b^2+ab}{a^2+b^2-ab}$

$b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Solve ⇒
I

$a = \frac{1}{b}$, $b = \frac{1}{a}$

$$\frac{a^2+\frac{1}{a^2}+1}{a^2+\frac{1}{a^2}-1} = ?$$

$$a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$a = (\sqrt{3}+\sqrt{2})^2$$

$$\boxed{\begin{array}{l} a \Rightarrow 5+2\sqrt{6} \\ b \text{ or } \frac{1}{a} \Rightarrow 5-2\sqrt{6} \end{array}}$$

$$\boxed{a+\frac{1}{a} = 10}$$

$$\downarrow$$

$$\boxed{\frac{99}{97}}$$ Ans

~~8~~

3 $a = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$, $b = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ find ⇒ $\frac{a^2+b^2+ab}{a^2+b^2-ab}$

Solve $\frac{a^2+\frac{1}{a^2}+1}{a^2+\frac{1}{a}-1} = ?$

$a = \frac{8+2\sqrt{15}}{2} \Rightarrow 4+\sqrt{15}$

$b \text{ or } \frac{1}{a} = 4-\sqrt{15}$

$a+\frac{1}{a} = 8$

$$\downarrow$$

$$\boxed{\frac{63}{61}}$$

4. $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, $b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ find $\frac{a}{b} + \frac{b}{a}$

Solve

$a = 5+2\sqrt{6}$

$\frac{1}{a}$ or $b = 5-2\sqrt{6}$

$a+\frac{1}{a} = 10$

$\frac{a}{b} + \frac{b}{a} \Rightarrow \frac{a^2+b^2}{ab}$

$\Rightarrow \frac{a^2+\frac{1}{a^2}}{1}$

$$\boxed{a^2+\frac{1}{a^2} = 98}$$

5. $a = \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$, $b = \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$ find $\frac{a}{b} + \frac{b}{a}$

Solve

$a = 6+\sqrt{35}$

$\frac{1}{a}$ or $b = 6-\sqrt{35}$

$a+\frac{1}{a} = 12$

$$\boxed{a^2+\frac{1}{a^2} \Rightarrow 142}$$

Next Type :→

1. $a = 3+2\sqrt{2}$

find $\frac{a^6+a^4+a^2+1}{a^3}$

Solve

$\frac{a^6}{a^3} + \frac{a^4}{a^3} + \frac{a^2}{a^3} + \frac{1}{a^3}$

$a^3 + a + \frac{1}{a} + \frac{1}{a^3}$

$a^3 + \frac{1}{a^3} + a + \frac{1}{a} \Longrightarrow \boxed{198+6 = 204}$

$a = 3+2\sqrt{2}$

$\frac{1}{a} = 3-2\sqrt{2}$

$a+\frac{1}{a} = 6$

$a^3+\frac{1}{a^3} = 198$

2 $a = 2+\sqrt{3}$ find $\dfrac{a^6+a^5+a^4+a^2+a+1}{a^3}$

Solve

$a = 2+\sqrt{3}$

$\frac{1}{a} = 2-\sqrt{3}$

$\boxed{a+\frac{1}{a} = 4}$

$\boxed{a^3+\frac{1}{a^3} = 52}$

$\boxed{a^2+\frac{1}{a^2} = 14}$

$\Rightarrow a^3+a^2+a+\frac{1}{a}+\frac{1}{a^2}+\frac{1}{a^3}$

$\Rightarrow a^3+\frac{1}{a^3}+a^2+\frac{1}{a^2}+a+\frac{1}{a}$

$52+14+4 \Rightarrow 70$ Ans

Bar (बार) [Recurssion] →

Trick Type→1

1⇒ $.\overline{2} = \frac{2}{9}$

2⇒ $.\overline{37} = \frac{37}{99}$

3⇒ $.\overline{476} = \frac{476}{999}$

Type→2

1⇒ $.2\overline{3} \Rightarrow \frac{23-2}{90} = \frac{21}{90}$

2⇒ $.3\overline{7} \Rightarrow \frac{37-3}{90} \Rightarrow \frac{34}{90}$

3⇒ $.4\overline{38} \Rightarrow \frac{438-4}{990} \Rightarrow \frac{434}{990}$

Type→III

1. $\overline{5}.40 \Rightarrow -5+.40 \Rightarrow -4.60$
2. $\overline{9}.60 \Rightarrow -9+.60 \Rightarrow -8.40$
3. $\overline{12}.20 \Rightarrow -12+.20 \Rightarrow -11.80$

Type→4

1⇒ $.\overline{2}+.1\overline{2}+.1\overline{23}$

$.\overline{2} = .22222$

$.1\overline{2} = .12222$

$.1\overline{23} = .12323$

$.46767$

$.4\overline{67} = \frac{467-4}{990} = \frac{463}{990}$

2012 (M)

Q Find the Cube root of ⇒ $.\overline{037}$

Solve

$$\boxed{.\overline{037} = \frac{037}{9}}$$

$0.\overline{037} \Rightarrow \sqrt[3]{0.\overline{037}}$

$\Rightarrow \sqrt[3]{\frac{37}{999}}$

$\Rightarrow \sqrt[3]{\frac{1}{27}}$

$\Rightarrow \frac{1}{3}$ Ans

Next Type :→

★★★★

1 $x = a^{1/3} + a^{-1/3}$ find $x^3 - 3x$

Solve

$$x = \sqrt[3]{a} + \frac{1}{\sqrt[3]{a}}$$

Cube करने पर

$$x^3 = a + \frac{1}{a} + 3 \cdot \cancel{\sqrt[3]{a}} \cdot \frac{1}{\cancel{\sqrt[3]{a}}} [x]$$

$$\boxed{x^3 - 3x = a + \frac{1}{a}}$$

2 $x = 2^{1/3} + 2^{-1/3}$ find $x^3 - 3x$

Solve

$$x = \sqrt[3]{2} + \frac{1}{\sqrt[3]{2}}$$

$$x^3 = 2 + \frac{1}{2} + 3x$$

$$\boxed{x^3 - 3x = 2 + \frac{1}{2} \text{ or } 2\frac{1}{2}}$$

3 $x = 5^{1/3} + 5^{-1/3}$ find $5x^{-3} - 15x$

Solve

$$x = \sqrt[3]{5} + \frac{1}{\sqrt[3]{5}}$$

Cube

$$x^3 = 5 + \frac{1}{5} + 3x$$

$$x^3 - 3x = \frac{26}{5}$$

$$\boxed{5x^3 - 15x = 26}$$

4 $x = 3^{2/3} - 3^{-2/3}$ find $9x^3 + 27x$

Solve

$$\boxed{\cancel{x} = \cancel{Rx}\ \cancel{(\sqrt[3]{3})^2} - \frac{1}{(\sqrt[3]{3})^2}}$$

$$x = (3^2)^{1/3} - [(3)^2]^{-1/3}$$

$$x = \sqrt[3]{9} - \frac{1}{\sqrt[3]{9}}$$

$$x^3 = 9 - \frac{1}{9} - 3x$$

$$x^3 + 3x = \frac{80}{9}$$

$$\boxed{9x^3 + 27x = 80}$$

5 $x = 2 + 2^{2/3} - 2^{1/3}$ find $x^3 - 6x^2 + 18x$

Solve

$$x = 2 + (4)^{1/3} - \sqrt[3]{2}$$

$$\cancel{x = 2 + \sqrt[3]{4} - \sqrt[3]{2}}$$

$$x - 2 = \sqrt[3]{4} - \sqrt[3]{2}$$

$$x^3 - 8 -$$

$$(x-2) = 2^{2/3} - 2^{1/3}$$

Cube करने पर

$$x^3 - 8 - 6x(x-2) = 4 - 2 - 3.2^{2/3}.2^{1/3}(x-2)$$

$$x^3 - 8 - 6x^2 + 12x = 2 - 6(x-2)$$

$$\boxed{x^3 - 6x^2 + 18x = 22}$$

Next Type →

Imagination Rule

1- $$\frac{x-a}{b+c}+\frac{x-b}{c+a}+\frac{x-c}{a+b}=3$$ find = x

Solve

$$\overset{1}{\boxed{\frac{x-a}{b+c}}}+\overset{1}{\boxed{\frac{x-b}{c+a}}}+\overset{1}{\boxed{\frac{x-c}{a+b}}}=3$$

$$\frac{x-a}{b+c}=1$$

$$\boxed{x=a+b+c}$$

2 $$\frac{x+a}{b+c}+\frac{x+b}{c+a}+\frac{x+c}{a+b}=-3$$ find x = ?

Solve

$$\frac{x+a}{b+c}=-1$$

$$\boxed{x=-a-b-c}$$

3 $$\frac{x-a^2}{b^2+c^2}+\frac{x-b^2}{c^2+a^2}+\frac{x^2-c^2}{a^2+b^2}=3$$ find x = ?

Solve

$$\frac{x-a^2}{b^2+c^2}=1$$

$$\boxed{x=a^2+b^2+c^2}$$

4 $\frac{x^2-a}{b+c}+\frac{x^2-b}{c+a}+\frac{x^2-c}{a+b}=3$ find $x=?$

Solve

$$\frac{x^2-a}{b+c}=1$$

$$x^2=a+b+c$$

$$\boxed{x=\sqrt{a+b+c}}$$

Next Type

Based On +1, -1

1 $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$ find $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=?$

Solve

$$\frac{a}{1-a}+1+\frac{b}{1-b}+1+\frac{c}{1-c}+1=1+3$$

$$\boxed{\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=4}$$

2 $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=3$ find $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=?$

Solve

$$\frac{1}{1-a}-1+\frac{1}{1-b}-1+\frac{1}{1-c}-1=0$$

$$\boxed{\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=0}$$

31/12/16

3 $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}=1$, find $\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}=?$

2014

Solve

(I)

$\boxed{a+b+c=0}$

$$\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}=\frac{a^{\cancel{2}}}{\cancel{-a}}+\frac{b^{\cancel{2}}}{\cancel{-b}}+\frac{c^{\cancel{2}}}{\cancel{-c}}$$

$$=-a-b-c$$

$$=-[a+b+c) \Rightarrow 0 \text{ Ans}$$

Maths by Puneet chaudhary sir

★ $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}=1$ find $a+b+c$

Solve

$$\overset{1/3}{\boxed{\frac{a}{b+c}}}+\overset{1/3}{\boxed{\frac{b}{c+a}}}+\overset{1/3}{\boxed{\frac{c}{a+b}}}$$

$$\frac{a}{b+c}=\frac{1}{3} \;\Big|\; \frac{b}{c+a}=\frac{1}{3} \;\Big|\; \frac{c}{a+b}=\frac{1}{3}$$

$$3a=b+c \quad 3b=c+a \quad 3c=a+b$$

$$3a+3b+3c=2a+2b+2c$$

$$\boxed{a+b+c=0} \text{ Ans}$$

3 > II

Solve

$$\frac{[a+b+c]}{[a+b+c]}\left[\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}=1\right]$$

$$\frac{a^2+a(b+c)}{b+c}+\frac{b^2+b(a+c)}{c+a}+\frac{c^2+c(a+b)}{a+b}=a+b+c$$

$$\frac{a^2}{b+c}+\cancel{\frac{a(b+c)}{b+c}}+\frac{b^2}{c+a}+\cancel{\frac{b(a+c)}{a+c}}+\frac{c^2}{a+b}+\cancel{\frac{c(a+b)}{a+b}}=a+b+c$$

$$\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}=\cancel{a}+\cancel{b}+\cancel{c}-\cancel{a}-\cancel{b}-\cancel{c}$$

$$\boxed{\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}=0}$$

Next Type

(1) $(x-2)(x-3)=1$, find $(x-2)^3-\frac{1}{(x-2)^3}$

Solve

$$(x-3)+1=\frac{1}{(x-2)}+1$$

$$(x-2)-\frac{1}{(x-2)}=1$$

$$(x-2)^3-\frac{1}{(x-2)^3}=4$$

Ans

$$\left[\begin{array}{l} x-\frac{1}{x}=k \\ x^3-\frac{1}{x^3}=k^3+3k \end{array}\right.$$

2 $(x-4)(x-3)=1$, $(x-3)^3-\frac{1}{(x-3)^3}$

Solve

$$(x-4)+1=\frac{1}{(x-3)}+1$$

$$(x-3)-\frac{1}{(x-3)}=1$$

$$(x-3)^3-\frac{1}{(x-3)^3}=4$$

Ans

(3) $(x-7)(x-3)=1$, $(x-3)^3 - \frac{1}{(x-3)^3} = ?$

Solve

$$(x-7)+4 = \frac{1}{(x-3)} + 4$$

$$(x-3) - \frac{1}{(x-3)} = 4$$

$$\boxed{(x-3)^3 - \frac{1}{(x-3)^3} = 64+12 = 76}$$

Ans

(4) $a-b+4=0$ & $(x-a)(x-b)=1$ find $(x-a)^3 - \frac{1}{(x-a)^3} = ?$

Solve $a-b+4=0$

$$\boxed{b=a+4}$$

$$(x-a)(x-b)=1$$

$$(x-b) = \frac{1}{(x-a)}$$

$$[x-a]-4 = \frac{1}{(x-a)}$$

$$(x-a) - \frac{1}{(x-a)} = 4$$

$$\boxed{(x-a)^3 - \frac{1}{(x-a)^3} = 76}$$

Ans

(5) $b-a+6=0$ $(x-a)(x-b)=1$ find $(x-b)^3 - \frac{1}{(x-b)^3} = ?$

Solve $a=b+6$

$$(x-a) = \frac{1}{(x-b)}$$

$$(x-b) - \frac{1}{(x-b)} = 6$$

$$(x-b)^3 - \frac{1}{(x-b)^3} = 216+18$$

$$\boxed{(x-b)^3 - \frac{1}{(x-b)^3} = 234}$$

Ans

6 $a-b+4=0$ & $(x-a)(x-b)=1$ find $(x-a)^3+\frac{1}{(x-a)^3}=?$

Solve

$b=a+4$ $\quad (x-b)=\frac{1}{(x-a)}$

$$\left[\begin{array}{l} x-\frac{1}{x}=K \\ x+\frac{1}{x}=\sqrt{K^2+4} \end{array}\right.$$

$$[x-a-4]-\frac{1}{x-a}=0$$

$$(x-a)-\frac{1}{(x-a)}=4$$

$$(x-a)+\frac{1}{(x-a)}=\sqrt{20}\Rightarrow 2\sqrt{5}$$

$$(x-a)^3+\frac{1}{(x-a)^3}=(2\sqrt{5})^3+3\times2\sqrt{5}$$

$$=40\sqrt{5}-6\sqrt{5}$$

$$=34\sqrt{5}$$ Ans

7 $b-a+6=0$ $\quad (x-a)(x-b)=1$ $\quad (x-b)^3+\frac{1}{(x-b)^3}=?$

Solve

$a=b+6$ $\quad (x-a)=\frac{1}{(x-b)}$

$$(x-b)-\frac{1}{(x-b)}=6$$

$$(x-b)+\frac{1}{(x-b)}=\sqrt{40}=2\sqrt{10}$$

$$(x-b)^3+\frac{1}{(x-b)^3}=(2\sqrt{10})^3-3\times2\sqrt{10}$$

$$=80\sqrt{10}-6\sqrt{10}$$

$$=74\sqrt{10}$$ //Ans

Imagination Rule :→

Type-I

If $x+\frac{1}{x}=2$

It is possible when $x=1$

$x+\frac{1}{x}=2$

$x^2+1=2x$

$x^2+1-2x=0$

$\boxed{x^2-1=0}$

$x-1=0$

$x=1$

1 $x^{70}+\frac{1}{x^{70}} \Rightarrow 2$

2 $x^{60}+x^{30}+x^{10}+1 \Rightarrow 4$

3 $x^{1}+\frac{1}{x^{72}} \Rightarrow 2$

4 $x^{1024}+\frac{1}{x^{1024}}=2$

Type → II

$x+\frac{1}{x}=-2$ It is posible when $\Rightarrow x=-1$

① $x^{60}+x^{50}+x^{40}+x^{30}+x^{20}+x^{10}+x \Rightarrow 6-1=5$

② $x^{60}+\frac{1}{x^{60}} \Rightarrow 2$

③ $x^{60}+\frac{1}{x^{61}} \Rightarrow 0$

Type → III

1→ if $x+\frac{4}{x}=4$, $x\neq 0$ find ① $x^2+\frac{1}{x^3}$

② $x^4+x^3+x^2+x$

Solve

① $x+\frac{4}{x}=4$ ✓

Put $x=2$ $\qquad x^2+\frac{1}{x^3}=4+\frac{1}{8}=\frac{33}{8}$

II $\quad x^4+x^3+x^2+x \Rightarrow 16+8+4+2 \Rightarrow 30$

2 $\quad x+\frac{4}{x}=-4$ $\quad$ find ① $x+\frac{1}{x^2}=?$ $\quad$ (ii) $x^3+\frac{1}{x^2}=?$

Solve

Put $x=-2$ $\qquad -2+\frac{1}{4}=-8+1=\underline{-7}$

Type → IV

1 If $\quad a+b+c=6$

$a^2+b^2+c^2=14$ $\qquad$ find $abc=?$

$a^3+b^3+c^3=36$

Solve

$a=1,\ b=2,\ c=3$ $\quad$ [Condition Satisfied होनी चाहिए]

$abc=1\times2\times3=6$ ~~Ans~~

2 If $\quad a+b+c=2$ $\qquad$ find $(a+b)^2+(b+c)^2+(c+a)^2$

$ab+bc+ca=-1$

Solve

$a=2,\ b=1,\ c=1$

$(2+1)^2+(1-1)^2+(-1+2)^2 \Rightarrow 9+0+1 \Rightarrow 10$ Ans

Type → V

☆☆☆☆

1. If abc are non Zero real no's such that $a+b+c=0$, $b^2 \neq ca$ find $\frac{a^2+b^2+c^2}{b^2-ca}$

Solve

I $a+b+c=0$, $b^2 \neq ca$

$a+c=-b$

Sq

$a^2+c^2+2ac=b^2$

$a^2+c^2=b^2-2ac$

$$\frac{a^2+b^2+c^2}{b^2-ca}$$

$$\frac{2b^2-2ac}{b^2-ca} \Rightarrow 2\left[\frac{\cancel{b^2-ac}}{\cancel{b^2-ac}}\right]$$

$\Rightarrow 2$

H & T

II $a+b+c=0$

$a=1$
$b=1$
$c=-2$

$$\frac{a^2+b^2+c^2}{b^2-ca} \Rightarrow \frac{1+1+4}{1+2} = \frac{\cancel{6}}{\cancel{3}} = 2 \text{ A}$$

2. $\sqrt{a}+\sqrt{b}+\sqrt{c}=0$ find $\frac{(a+b-c)^2}{ab}$

H & T

$\sqrt{a}=1$ | $a=1$
$\sqrt{b}=1$ | $b=1$
$\sqrt{c}=-2$ | $c=4$

$$\frac{(a+b-c)^2}{ab} \Rightarrow \frac{(1+1-4)^2}{1\times1}$$

$\Rightarrow 4$ Ans

Type → VI

1. If $x^2 = y + z$
$y^2 = z + x$
$z^2 = x + y$
find $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = ?$

Ⓘ **Solve** H & T
$x = y = z = 2$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$ An

Ⓘⓘ $\frac{x}{x}\left[\frac{1}{x+1}\right] + \frac{y}{y}\left[\frac{1}{y+1}\right] + \frac{z}{z}\left[\frac{1}{z+1}\right]$

$\frac{x}{x^2+x} + \frac{y}{y^2+y} + \frac{z}{z^2+z}$

$\frac{x}{x+y+z} + \frac{y}{x+y+z} + \frac{z}{x+y+z} = \cancel{\frac{x+y+z}{x+y+z}}$

$= 1$ An

2. $\frac{x^2}{by+cz} = \frac{y^2}{cz+ax} = \frac{z^2}{ax+by} = 1$ find $\frac{a}{a+x} + \frac{b}{b+y} + \frac{c}{c+z} = ?$

H & T Ⓘ
$a = b = c = 1$
$x = y = z = 2$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$ A

Ⓘⓘ $x^2 = by + cz$, $y^2 = cz + ax$, $z^2 = ax + by$

$\frac{x}{x}\left[\frac{a}{a+x}\right] + \frac{y}{y}\left[\frac{b}{b+y}\right] + \frac{z}{z}\left[\frac{c}{c+z}\right]$

$\frac{ax}{ax+x^2} + \frac{by}{by+y^2} + \frac{cz}{cz+z^2} \Rightarrow \frac{ax}{ax+by+cz} + \frac{by}{ax+by+cz} + \frac{cz}{ax+by+cz}$

$\cancel{\frac{ax+by+cz}{ax+by+cz}} = 1$ A

★ Type - VI

1. If $x+\frac{1}{y}=1$, $y+\frac{1}{z}=1$ find $= z+\frac{1}{x}=?$

I

$x=2$

$\boxed{y=-1}$

$z=\frac{1}{2}$

$z+\frac{1}{x}$

$\Rightarrow \frac{1}{2}+\frac{1}{2}=\frac{\cancel{2}}{\cancel{2}}=1$ Ans

2. $a+\frac{1}{b}=1$, $b+\frac{1}{c}=1$ find $abc=?$

$a=2$

$\boxed{b=-1}$

$c=\frac{1}{2}$

$abc=\cancel{2}\times-1\times\frac{1}{\cancel{2}}=-1$ Ans

II

$x+\frac{1}{y}=1$

$x=1-\frac{1}{y}$

$\boxed{\frac{1}{x}=\frac{y}{y-1}}$

$y+\frac{1}{z}=1$

$\frac{1}{z}=1-y$

$\boxed{z=\frac{1}{1-y}}$

$z+\frac{1}{x}$

$\Rightarrow \frac{1}{1-y}+\frac{y}{y-1}$

$\Rightarrow \frac{1}{1-y}-\frac{y}{1-y}$

$\Rightarrow \frac{\cancel{1-y}}{\cancel{1-y}}=1$ Ans

Type → VII

1. If $2s=(a+b+c)$

find $(s-a)^2+(s-b)^2+(s-c)^2+s^2-a^2-b^2-c^2=?$

Solve

$$(s-a)^2+(s-b)^2+(s-c)^2+s^2-a^2-b^2-c^2$$

$$s^2+\cancel{a^2}-2as+s^2+\cancel{b^2}-2bs+s^2+\cancel{c^2}-2cs+s^2-\cancel{a^2}-\cancel{b^2}-\cancel{c^2}$$

$$4s^2-2s(a+b+c)$$

$$4s^2-2s\times 2s$$

$$\cancel{4s^2}-\cancel{4s^2}=0$$

H & T

$$\left\{\begin{array}{r} a=b=c=2 \\ s=3 \end{array}\right\}$$

$$1+1+1+9-4-4-4 \Rightarrow \cancel{12}-\cancel{12} \Rightarrow 0$$ **Ans**

2 If $a^m.a^n=a^{mn}$ find $m(n-2)+n(m-2)$

Solve

I

$$a^{m+n}=a^{mn}$$

$$\boxed{m+n=mn}$$ ✓

Put $m=n=2$

$$m(2-2)+n(2-2)$$

$$0+0=0$$

II

$$mn-2m+mn-2n$$

$$2mn-2[m+n]$$

$$\cancel{2mn}-\cancel{2mn}=0$$

3

$x(x+y+z)=20$

$y(x+y+z)=30$

$z(x+y+z)=50$

find $2(x+y+z)=?$

★ **Solve**

I $x = 2$

$y = 3$ $2(x+y+z)$

$z = 5$ $2 \times 10 = 20$ Ans

4 $a^3 b = abc = 180$ find $c = ?$

(I) $a^{\cancel{3}} \cancel{b} = \cancel{a}\cancel{b}c$

$\boxed{a^2 = c}$

$a = c$

$a = 1$ $\boxed{c = 1}$ Ans

=

5 $x^3 y = xyz = 150$ find $z = ?$

(II) $x = 1$

$z = 1$ $1 \times 150 = 1 \times 150 \times 1 = 150$

$y = 150$ $\boxed{150 = 150 = 150}$

2015

6 $a^x = (x+y+z)^y$

$a^y = (x+y+z)^z$ find $(x+y+z) = ?$

$a^z = (x+y+z)^x$

Solve $a^{\cancel{x+y+z}} = (x+y+z)^{\cancel{x+y+z}}$

$\boxed{a = x+y+z}$

7 If $m=a^x$, $n=a^y$ $m^y n^x = a^{2/z}$ find $= xyz$

Solve

$$a^{xy} . a^{xy} = a^{2/z}$$

$$a^{2xy} = a^{2/z}$$

$$\cancel{2}xy = \frac{\cancel{2}}{z}$$

$$\boxed{xyz=1}$$

8 ~~If~~ $x = b+c-2a$

$y = c+a-2b$

$z = a+b-2a$

find $x^2+y^2-z^2+2xy$

$\downarrow$

Solve

$$x = \cancel{b}+\cancel{c}-\cancel{2a}$$

$$y = \cancel{c}+\cancel{a}-\cancel{2b}$$

$$z = \cancel{a}+\cancel{b}-\cancel{2a}$$

$$x+y+z=0$$

$\Rightarrow (x+y)^2 - z^2$

$\Rightarrow (x+y+z)(x+y-z)$

$\Rightarrow 0\ (x+y-z)$

$\Rightarrow 0$ Ans

9 $x=y^a$, $y=z^b$, $z=x^c$ find $abc=?$

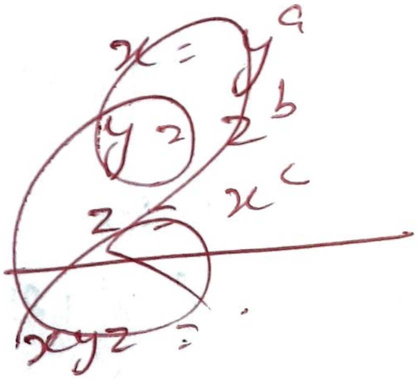
Solve

$x=y^a$, $y=z^b$ $z=x^c$

$y=(x^c)^b$ $y=x^{bc}$

$x = x^{abc}$

$\boxed{abc=1}$



Next Type

07/01/17

① If $x = 70$ — **even Power**

find $x^4 - 71x^3 + 71x^2 - 71x + 75$

$-70 + 75 \Rightarrow 5$ **Ans**

Solve

$x = 70 \Rightarrow x - 70 = 0$

$\Rightarrow x^4 - 70x^3 - x^3 + 70x^2 + x^2 - 70x - x + 70 + 5$

$\Rightarrow x^3(x - 70) - x^2(x - 70) + x(x - 70) - 1(x - 70) + 5$

$\Rightarrow 0 - 0 + 0 - 0 + 5$

$\Rightarrow 5$ **Ans**

② If $x = 12$

find $x^4 - 13x^3 + 13x^2 - 13x + 15$

$-12 + 15 = 3$ **Ans**

Solve

$x - 12 = 0$

$= x^4 - 12x^3 - x^3 + 12x^2 + x^2 - 12x - x + 12 + 3$

$= x^3(x - 12) - x^2(x - 12) + x(x - 12) - (x - 12) + 3$

$= 0 \quad 0 \quad 0 \quad 0 \quad 3$ **Ans**

3 if $x = 25$ — **odd Power**

find $x^5 - 26x^4 + 26x^3 - 26x^2 + 26x - 30$

$25 - 30 = -5$ **Ans**

Solve

$x - 25 = 0$

$\Rightarrow x^5 - 25x^4 - x^4 + 25x^3 + x^3 - 25x^2 - x^2 + 25x + x - 25 - 5$

$\Rightarrow x^4(x - 25) - x^3(x - 25) + x^2(x - 25) - x(x - 25) + 1(x - 25) - 5$

$0 \quad 0 \quad 0 \quad 0$

$= -5$ **Ans**

Maxima & Minima

① find min value of

$3x^2-6x+1$

Solve

I

$\frac{\text{Min}}{\text{diff}}$ $3x^2-6x+1$ $\left[\overset{\times}{3}x^{2}-\overset{\times}{6}x^{1}+1\right]$

$6x-6=0$

$6x=6$

$\boxed{x=1}$

$3-6+1 \cancel{=0}$

$4-6=\cancel{0}-2$ Ans

II

$\frac{4ac-b^2}{4a}$

$\frac{4\times3\times1-36}{4\times3} \Rightarrow \frac{12-36}{12} \Rightarrow \frac{\cancel{-24}}{\cancel{12}}$

$\Rightarrow -2$ Ans

For ax^2+bx+c

Case I

when $a>0$

then expression gives min value

$\Rightarrow \frac{4ac-b^2}{4a}$

Case 2 →

when $a<0$

exp gives max value

$=\frac{4ac-b^2}{4a}$

2 Find max value of

$13-4x-x^2$

Solve (I)

$-4-2x=0$

$-2x=4$

$\boxed{x=-2}$

$13+8-4$

$21-4$

17 Ans

II

$\frac{4ac-b^2}{4a}$

$\frac{4\times(-1)\times13-16}{4\times(-1)}$

$\frac{-52-16}{-4}=\frac{\cancel{+68}}{\cancel{+4}}$

$=17$ // Ans

3 Find min value of $4x^2-16x+17$

Solve

diff

$$8x-16=0$$

$$\boxed{x=2}$$

$$16-32+17$$

$$33-32=1$$ Ans

4 find min value

(i) $(9-x)(2-x)$

(ii) $(x+3)(x-6)$

Solve

I

$$(9-x)(2-x)$$

$$18-9x-2x+x^2$$

$$x^2-11x+18$$

$$\frac{4ac-b^2}{4a} \Rightarrow \frac{4\times1\times18-121}{4}$$

$$\Rightarrow \frac{72-121}{4}$$

$$\Rightarrow \frac{-49}{4}$$ Ans

II

$$(x+3)(x-6)$$

$$\Rightarrow x^2-6x+3x-18$$

$$\Rightarrow x^2-3x+18$$

$$\frac{4ac-b^2}{4a}$$

$$\frac{4\times1\times18-(-3)^2}{4\times1}$$

$$\frac{72-9}{4}=\frac{63}{4}$$ Ans

Type-2

① Find Min value

$(x-5)^2+51$

Solve

$x^2+25-10x+51$

$x^2-10x+76$

$\frac{4\times1\times76-100}{4} \Rightarrow \frac{304-100}{4}$

$\Rightarrow \frac{\cancel{204}}{\cancel{4}} = 51$ Ans

min will be 0 when

$x-5=0$

then

$0+51 \Rightarrow 51$ Ans

2 $(5x+2)^2+21$

Solve

$(5x+2)^2+21$

$0+21 = 21$ Ans

Type→3

① find min value of

$x^2+\frac{1}{x^2}-3$

Solve $x^2+\frac{1}{x^2}$

$\boxed{x=1}$ $x^2+\frac{1}{x^2} = 2$

Min value 2 ही होगी |

$2-3 = -1$

② find min value of

$$x^2+\frac{1}{x^2}-7$$

Solve

$$\downarrow$$

$$2-7=-5$$ Ans

③ find min value of

$$x^2+\frac{1}{x^2+1}-3$$

Solve

$$x^2+1+\frac{1}{x^2+1}-3-1$$

$$\downarrow$$

$$2-4\Rightarrow -2$$ Ans

Tukka →

$x=0$

$$x^2+\frac{1}{x^2+1}-3$$

$$0+\frac{1}{0+1}-3\Rightarrow 1-3$$

$$\Rightarrow -2$$ Ans

Type→4

① If $x+y=6$ find the max value of xy

Solve $x+y=6$

Max $\boxed{\text{When } x=y}$

$x=3$

$y=3$

$x\cdot y=3\times 3$

$=9$ Ans

2 If $a+b+c+d=1$ find max value of $ab+bc+cd+da$

Solve

$$a=b=c=d=\frac{1}{4}$$

$$\Rightarrow \frac{1}{4}\times\frac{1}{4}+\frac{1}{4}*\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}*\frac{1}{4}$$

$$\Rightarrow \frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16} \Rightarrow \frac{4}{16} \Rightarrow \boxed{\frac{1}{4}}$$ Ans

3 If $a+b+c+d=1$

find the max value of $(1+a)(1+b)(1+c)(1+d)$

Solve

$$a=b=c=d=\frac{1}{4}$$

$$\Rightarrow \left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{4}\right)$$

$$\Rightarrow \left(\frac{5}{4}\right)^4$$ Ans

Next Type

Formula $a^3+b^3+c^3-3abc=(a+b+c)\underline{(a^2+b^2+c^2-\cancel{2}ab-\cancel{2}bc-ca)}$

if $a+b+c=0$

$$a^3+b^3+c^3-3abc=0$$

$$\boxed{a^3+b^3+c^3=3abc}$$

$$\Rightarrow \frac{2}{2}\left[a^2+b^2+c^2-ab-bc-ca\right]$$

$$\Rightarrow \frac{1}{2}\left[2a^2+2b^2+2c^2-2ab-2bc-2ca\right]$$

$$\Rightarrow \frac{1}{2}\left[a^2+a^2+b^2+b^2+c^2+c^2-2ab-2bc-2ca\right]$$

$$\Rightarrow \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

① If $a = 99$, $b = 100$, $c = 101$, $a^2+b^2+c^2-ab-bc-ca = ?$

Solve

$\Rightarrow \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

$\Rightarrow \frac{1}{2}\left[1+1+4\right]$

$\Rightarrow \frac{6}{2} = 3$ Ans.

P = diff

$3p^2$

$3 \times 1 = 3$ Ans

2 If $a = 998$, $b = 1000$, $c = 1002$ $\left[a^2+b^2+c^2-ab-bc-ca\right]$

Solve

$\Rightarrow \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-b)^2\right]$

$\Rightarrow \frac{1}{2}\left[4+4+16\right]$

$\Rightarrow \frac{24}{2} = 12$

$3p^2$

$3 \times 4 = 12$ Ans

3 If $a = 997$, $b = 1000$, $c = 1003$ $\left[a^2+b^2+c^2-ab-bc-ca\right]$

$3p^2$

Solve $\frac{1}{2}\left[9+9+36\right) \Rightarrow \frac{54}{2} = 27$

$3 \times 9 = 27$

Type B

① if $a = 23$
$b = 24$
$c = 25$

$a^3+b^3+c^2-3abc = ?$

$\Downarrow$

Solve

$\underline{(a+b+c)}\left[a^2+b^2+c^2-ab-bc-ca\right]$

$\Rightarrow (a+b+c)\ 3p^2$

$\Rightarrow 72 \times 3 \Rightarrow 216$ Ans

2 If $a = 997$
$b = 1000$
$c = 1003$

find $a^3+b^3+c^2-3abc = ?$

Solve

$\Rightarrow (a+b+c)\ 3p^2$

$\Rightarrow 3000 \times 3 \times 9$

$\Rightarrow 9000 \times 9 \Rightarrow 81000$ Ans

3 $a = 99$
$b = 100$
$c = 101$

find $a^3+b^3+c^3-3abc = ?$

$\Downarrow$

Solve

$\Rightarrow (a+b+c)\ 3p^2$

$\Rightarrow 300 \times 3$

$\Rightarrow 900$ Ans

Type→C

4

$a = 19$

$b = 20$

$c = 21$

find $\dfrac{a^3+b^3+c^3-3abc}{(a+b+c)}$

Solve

$$\frac{\cancel{(a+b+c)}\,3p^2}{\cancel{(a+b+c)}} \Rightarrow 3 \times 1 = 3 \text{ Ans}$$

5

$a = 98$

$b = 100$

$c = 102$

find $\dfrac{a^3+b^3+c^2-3abc}{a+b+c}$

Solve

$$\frac{\cancel{(a+b+c)}\,3p^2}{\cancel{(a+b+c)}} \Rightarrow 3 \times 4 = 12 \text{ Ans}$$

Type-D

① If $a+b+c = 0$

find $\dfrac{a^2}{bc} + \dfrac{b^2}{ca} + \dfrac{c^2}{ab} = ?$

Solve

$$\frac{a^3+b^3+c^3}{abc} \Rightarrow \frac{3abc}{\cancel{abc}} = 3 \text{ Ans}$$

② If $a+b+c=0$

$a^3+b^3+c^3+3abc = 0$

Solve $3abc + 3abc = 6abc$ Ans

3 if $\frac{(a-b)^2}{(b-c)(c-a)}+\frac{(b-c)^2}{(a-b)(c-a)}+\frac{(c-a)^2}{(b-c)(a-b}$

let $x=a-b$
$y=b-c$
$z=c-a$

$$\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a}=\frac{3\cancel{(a-b)(b-c)(c-a)}}{\cancel{(a-b)(b-c)(c-a)}}$$

$=3$ Ans

Or

$$\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}\Rightarrow\frac{x^3+y^3+z^3}{xyz}=\frac{3\cancel{xyz}}{\cancel{xyz}}$$

$=3$ Ans

4 if $(a-b)^3+(b-c)^3+(c-a)^3$

Solve

$\cancel{a}-\cancel{b}+\cancel{b}-\cancel{c}+\cancel{c}-\cancel{a}=0$

$3(a-b)(b-c)(c-a)$ Ans

5 if $(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3$

Solve

$3(x^2-y^2)(y^2-z^2)(z^2-x^2)$ Ans

6 if $\frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$

$$\Rightarrow\frac{\cancel{3}(x^2-y^2)(y^2-z^2)(z^2-x^2)}{\cancel{3}(x-y)(y-z)(z-x)}\Rightarrow\frac{(x+y)\cancel{(x-y)}(y+z)\cancel{(y-z)}(z+x)(\cancel{z}}{\cancel{(x-y)}\cancel{(y-z)}\cancel{(z-x)}}$$

$=(x+y)(y+z)(z+x)$ Ans

7

$x = a(b-c)$, $y = b(c-a)$, $z = c(a-b)$

find $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$

Solve

$$\frac{x}{a} = \cancel{b} - \cancel{c}$$

$$\frac{y}{b} = \cancel{c} - \cancel{a}$$

$$\frac{z}{c} = \cancel{a} - \cancel{b}$$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$$

~~$(b-c)^3 + (c-a)^3$~~

$$\Rightarrow 3\left[\frac{x}{a}\right]\left[\frac{y}{b}\right]\left[\frac{z}{c}\right]$$

$$\Rightarrow \frac{3xyz}{abc}$$ Ans

Next Type

① $\dfrac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1}$

$a^3 + b^3 = [a^2 + b^2 - ab][a+b]$

$a^3 - b^3 = (a^2 + b^2 + ab)(a-b)$

Solve

$$\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1}$$

$a^2 + ab + b^2$

$(\sqrt[3]{2})^3 + \sqrt[3]{2} \times 1 + 1$

$$\frac{1}{(\sqrt[3]{2})^2 + \sqrt[3]{2} \times 1 + 1^2} \times \frac{\sqrt[3]{2} - 1}{\sqrt[3]{2} - 1} \Rightarrow \frac{\sqrt[3]{2} - 1}{(\sqrt[3]{2})^3 - 1}$$

$(a^2 \quad ab \quad b^2)\ (a-b) \Rightarrow \sqrt[3]{2} - 1$

Ans

2 $$\frac{1}{\sqrt[3]{9}+\sqrt[3]{3}+1}$$

Solve

$$\frac{1}{\sqrt[3]{9}+\sqrt[3]{3}+1} \times \frac{\sqrt[3]{3}-1}{\sqrt[3]{3}-1}$$

$$\frac{\sqrt[3]{3}-1}{(\sqrt[3]{3})^3-1} \Rightarrow \frac{\sqrt[3]{3}-1}{2} \quad \text{Ans}$$

3 $$\frac{1}{\sqrt[3]{9}-\sqrt[3]{3}+1}$$

$$\left\{ \begin{array}{l} \dfrac{1}{\sqrt[3]{n^2}+\sqrt[3]{n}+1} = \dfrac{\sqrt[3]{n}-1}{(\sqrt[3]{n})^3-1} \\ \\ \dfrac{1}{\sqrt[3]{n^2}-\sqrt[3]{n}+1} = \dfrac{\sqrt[3]{n}+1}{(\sqrt[3]{n})^3+1} \end{array} \right.$$

Solve

$$\frac{\sqrt[3]{3}+1}{(\sqrt[3]{3})^3+1} \Rightarrow \frac{\sqrt[3]{3}+1}{4} \quad \text{Ans}$$

4 $$\frac{1}{\sqrt[3]{25}+\sqrt[3]{5}+1}$$

Solve

$$\frac{\sqrt[3]{5}-1}{(\sqrt[3]{5})^3-1} \Rightarrow \frac{\sqrt[3]{5}-1}{4} \quad \text{Ans}$$

5 $$\frac{1}{\sqrt[3]{25}-\sqrt[3]{5}+1} \Rightarrow$$

Solve

$$\frac{\sqrt[3]{5}+1}{(\sqrt[3]{5})^3+1} \Rightarrow \frac{\sqrt[3]{5}+1}{6} \quad \text{Ans}$$

Next Type

① $\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1} = a\sqrt[3]{4}+b\sqrt[3]{2}+c$ find a+b+c

$\Downarrow$

$0 + \sqrt[3]{2} - 1 = a\sqrt[3]{4} + b\sqrt[3]{2} + c$

$a = 0$

$b = 1$ $a+b+c = 0+\cancel{1}-\cancel{1}$

$c = -1$ $= 0$ Ans

$\sqrt{n+\sqrt{n-\sqrt{n+\sqrt{n \cdots\cdots\infty}}}}$

$\frac{\sqrt{4n-3}+1}{2}$ Ans

$\boxed{14/01/17}$

No of Zero

1 $15 \times 12 \times 14 \times 20 \times 35$

इस प्रकार के प्रश्नों में 5 और 2 के जोडे बनाते है।

Solve

$15 \times 12 \times 14 \times 20 \times 35$

$15 = 5 \times 3$

$12 = 3 \times 2^2$

$14 = 7 \times 2$

$20 = 2^2 \times 5$

$35 = 7 \times 5$

5 → 3 times

2 → 5 times

$\boxed{3 \text{ zero}}$ Ans

2 $1 \times 2 \times 3 \times 4 ----- 100$

Solve

5	100 ×
5	20
	4

2	100 ×
2	50
2	25
2	12
2	6
2	3
	1

5 → 24time
2 → 97 time

Zero → 24

3 $1 \times 2 \times 3 \times 4 ----- 200$

Solve

5	200 ×
5	40
5	8
	1

Zero → 49

4 $1 \times 2 \times 3 ------- 19$

Solve

5	19 ×
	3

Zero → 3

5- $1 \times 2 \times 3 ---- 99$

Solve

5	99 ×
5	19
	3

Zero → 22

Next Type

Odd / Even

Odd →

1. $1 \times 3 \times 5 \times ------ \times 99$
2. $1 \times 3 \times 5 \times ------ \times 1001$

odd में कोई zero नहीं आता है।

Even →

1. $2 \times 4 \times 6 \times 8 ------ 200$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \qquad\qquad \downarrow$$

$2\times1 \times 2\times2 \times 2\times3 \times 2\times4 \qquad 2\times100$

$2^{100}[1\times2\times3 --- 100]$

$\downarrow$

5 → 24 time

$\boxed{\text{Zero} \rightarrow 24}$

Ans

2. $2 \times 4 \times 6 \times 8 ------ 400$

$\downarrow$

2×200

$$\begin{array}{c|l} 5 & 200 \\ \hline 5 & 40 \\ \hline 5 & 8 \\ \hline & 1 \end{array}$$

$\boxed{\text{Zero} \rightarrow 49}$

1. $10 \times 20 \times 30 \times ----- \times 1000$

Solve

$$10 \times 20 \times 30 \times ---- \times 1000$$

$$\downarrow \quad \downarrow \quad \downarrow \qquad\qquad \downarrow$$

$$10\times1 \times 10\times2 \times 10\times3 \qquad 10\times100$$

$$\Rightarrow 10^{100}\underline{[1\times2\times3 ---- 100]}$$

$\rightarrow 100+24 \Rightarrow$ 124 zero

2. $10\times20\times30\times ----- \times 5000$

Solve

$$10 \times 20 \times 30 ------ \times 5000$$

$$\downarrow \quad \downarrow \quad \downarrow \qquad\qquad \downarrow$$

$$10\times1 \times 10\times2 \times 10\times3 ---- 10\times500$$

$$10^{500}[1\times2\times3 -- \times500]$$

5	500 ×
5	100
5	20
	4

$500+124 \rightarrow 624$

3. $7\times14\times21 ----- 700$

Solve

$$7\times14\times21 ---- 700$$

$$7^{100}\underbrace{(1\times2\times3 ---- 100)}$$

Ans = 24 zero

{7 कभी zero नहीं देता इसलिए Odd No के zero नहीं जोड़े जाते हैं।}

4 $5 \times 10 \times 15 \times 20 ----- \times 500$

Solve

$$5 \times 10 \times 15 \times 20 ----- \times 500$$

$$\Rightarrow 5 \times 1 \times 5 \times 2 \times 5 \times 3 ----- \times 5 \times 100$$

$$\Rightarrow 5^{100}[1 \times 2 \times 3 \times --- \times 100]$$

$$\Rightarrow 100 + 24 \Rightarrow 124 \text{ times} \leftarrow 5$$

$$97 \text{ times} \leftarrow 2$$

$$\boxed{\text{Zero} \rightarrow 97}$$

5 वाली संख्या में 2 के जोडे वाले zero होते है।

5 $\lfloor 100$ or $100!$

Solve

$$100 \times 97 \times 98 --- \times 1$$

$$1 \times 2 \times 3 \times 4 --- \times 100$$

$$\Rightarrow 24 \text{ zero}$$

6 $200!$

Solve

$$\begin{array}{r|l} 5 & 200 \\ \hline 5 & 40 \\ \hline 5 & 8 \\ \hline & 1 \end{array}$$

49 zero

$\underline{\underline{7}}$ $50! \times 100! \times 200!$

Solve

$$\begin{array}{ccccc} \lfloor\underline{50} & \times & \lfloor\underline{100} & \times & \lfloor\underline{200} \\ \downarrow & & \downarrow & & \downarrow \\ 12 & & 24 & & 49. \end{array}$$

इस प्रकार के प्रश्नों में गुणा वाले में Zero Add होगें |

$$12+24+49 \rightarrow \underline{\underline{85}}$$

$\underline{\underline{8}}$ $\lfloor\underline{50} + \lfloor\underline{100} + \lfloor\underline{200}$

Solve

$$\begin{array}{ccccc} \searrow & & \searrow & & \downarrow \\ \underline{\underline{12}} & + & 24 & + & 49 \end{array}$$

Addition वाले Que में कम वाला Ans होगें |

Ans $\underline{\underline{12}}$

$\underline{\underline{9}}$ $1^5 \times 2^5 \times 3^5 \times 4^5 \text{------} 30^5$

Solve

$$\left.\begin{array}{l} 5^5 \longrightarrow 5 \text{ time} \\ 10^5 (5\times2)^5 \longrightarrow 5 \text{ time} \\ 15^5 (5\times3)^5 \longrightarrow 5 \text{ time} \\ 20^5 (5\times4)^5 \longrightarrow 5 \text{ time} \\ 25^5 [5\times5]^5 \longrightarrow 10 \text{ time} \\ 30^5 [5\times6]^5 \longrightarrow 5 \text{ time} \end{array}\right\} \quad \underline{\underline{35 \text{ zero}}}$$

10 $1^1 \times 2^2 \times 3^3 \ldots\ldots 25^{25}$

Solve

$5^5 \rightarrow 5$

$10^{10} (5\times2)^{10} \rightarrow 10$

$15^{15} \rightarrow 15$

$20^{20} \rightarrow 20$

$25^{25} (25\times25)^{25} \rightarrow 50$

100 zero

11 If

$A = 1^2$

$B = 2^3$

$C = 3^4$

$D = 4^5$

$\vdots$

$Z = 26^{27}$

find the no of zero

$A \times B \times C \times D \ldots\ldots Z = ?$

Solve

$E \rightarrow 5^6 \rightarrow 6$

$J \rightarrow 10^{11} \rightarrow 11$

$O \rightarrow 15^{16} \rightarrow 16$

$T \rightarrow 20^{21} \rightarrow 21$

$Y \rightarrow 25^{26} \rightarrow 52$

106 zero

12 If $A = 1^3$, $B = 2^4$, $C = 3^5$ ----- $Z = 26^{29}$

$A \times B \times C \times D$ --- Z

$E \rightarrow 5^7 \rightarrow 7$
$J \rightarrow 10^{12} \rightarrow 12$
$O \rightarrow 15^{17} \rightarrow 17$
$T \rightarrow 20^{22} \rightarrow 22$
$Y \rightarrow 25^{27} \rightarrow 54$

$\overline{112}$ zero

Remainder Theorm

1 $\frac{7}{6} \Rightarrow 1$

2 $\frac{7^2}{6} \Rightarrow 1$

3 $\frac{7^{200}}{6} \Rightarrow \frac{(6+1)^{200}}{6} \Rightarrow 1$

Imp $\boxed{\text{Level} - 1}$

1 $\frac{25^{26}}{26} \Rightarrow \frac{(26-1)^{26}}{26} \Rightarrow +1$ Ans

2 $\frac{68^{68}}{67} \Rightarrow \frac{(67+1)^{68}}{67} \Rightarrow +1$ Ans

3 $\frac{18^{15}+4}{17} \Rightarrow \frac{(17+1)^{15}+4}{17} \Rightarrow 1+4 \Rightarrow 5$ Ans

4 $\frac{75^{65}+35}{76} \Rightarrow \frac{(76-1)^{65}+35}{76} \Rightarrow -1+35 \Rightarrow 34$ Ans

5 $\frac{85^{97}-27}{86} \Rightarrow \frac{(86-1)^{97}-27}{86} = -1-27=-28$

$86-28 \Rightarrow 58$ Ans

6 $\frac{7^{31}}{50} \Rightarrow \frac{7\cdot 7^{30}}{50} \Rightarrow \frac{7(7^2)^{15}}{50} \Rightarrow \frac{7(50-1)^{15}}{50} \Rightarrow -7$

$\Rightarrow 50-7=43$ Ans

7 $\frac{3^{23}}{5} \Rightarrow \frac{3.3^{22}}{5} \Rightarrow \frac{3(3^2)^{11}}{5} = \frac{3(10-1)^{11}}{5} \Rightarrow -3 \Rightarrow 5-3$

$\Rightarrow 2$ Ans

8 $\frac{67^{67}+67}{68} \Rightarrow \frac{(68-1)^{67}+67}{68} = -1+67 \Rightarrow 66$ Ans

9 $\frac{8^{64}+8}{9} \Rightarrow \frac{(9-1)^{64}+8}{9} \Rightarrow \frac{9}{9} \Rightarrow 0$ Ans

Level → 2

1 $\frac{17^{100}}{51} \Rightarrow \frac{\cancel{17}\cdot 17^{99}}{\cancel{51}_{3}} \Rightarrow \frac{17^{99}}{3} \Rightarrow \frac{(18-1)^{99}}{3} = -1$

$3-1=2\times 17=34$ Ans

2 $\frac{9^{50}}{36} \Rightarrow \frac{\cancel{9}\cdot 9^{49}}{\cancel{36}_{4}} \Rightarrow \frac{9^{49}}{4} \Rightarrow \frac{(8+1)^{49}}{4} \Rightarrow 1\times 9=9$ Ans

3. $\frac{3^{57}}{108} \Rightarrow \frac{\cancel{3}\cdot 3^{56}}{\cancel{108}_{36}} = \frac{3^{56}}{36} = \frac{\cancel{3}\cdot 3^{55}}{\cancel{36}_{12}} = \frac{3^{55}}{12} = \frac{\cancel{3}\cdot 3^{54}}{\cancel{12}_{4}}$

$\frac{3^{3}\cdot 3^{54}}{108} = \frac{\cancel{27}\times 3^{54}}{\cancel{108}_{4}} \Longrightarrow \frac{3^{54}}{4} \Rightarrow \frac{(4-1)^{54}}{4}$

$\Rightarrow 1\times 27 = 27$ Ans

Level-3 →✳← Rule of Cyclicity

1. $\frac{(72)^{10}}{7} \Rightarrow \frac{(2)^{10}}{7}$

$\left.\begin{aligned} \frac{2^{1}}{7} &= 2 \\ \frac{2^{2}}{7} &= 4 \\ \frac{2^{3}}{7} &= 1 \end{aligned}\right\}$ 3 cycle

$\frac{\text{Power}}{\text{Cycle}}$

$\Rightarrow \frac{\cancel{10}}{\cancel{3}} = 1$

$\frac{2^{1}}{7} = 2$ Ans

2. $\frac{(72)^{17}}{7} \Rightarrow \frac{2^{17}}{7}$

$\frac{2^{2}}{7} = 4$ Ans

3. $\frac{(72)^{50}}{7} \Rightarrow \frac{2^{50}}{7}$

$\frac{2^{2}}{7} = 4$ Ans

4 $\frac{(72)^{99}}{7} \Rightarrow \frac{2^{99}}{7}$

If Power is completly divided last turn of the cycle will be Ans.

1 Ans

5 $\frac{(73)^{50}}{7} \Rightarrow \frac{(3)^{50}}{7} = \frac{3^{2}}{7} \Rightarrow 2$ Ans

$$\left.\begin{array}{l} \frac{3^1}{7} = 3 \\ \frac{3^2}{7} = 2 \\ \frac{3^3}{7} = 6 \\ \hline \frac{3^4}{7} = 4 \\ \frac{3^5}{7} = 5 \\ \frac{3^6}{7} = 1 \end{array}\right\} \text{Cycle} \rightarrow 6$$

6 $\frac{(73)^{52}}{7} \Rightarrow \frac{(3)^{52}}{7} \Rightarrow \frac{(3)^{4}}{7} \Rightarrow \frac{3^{3}.3}{7} = \frac{6\times3}{7} = \frac{18}{7} = 4$ Ans

7 $\frac{19^{77}}{7} \Rightarrow \frac{5^{77}}{7} = \frac{5^{5}}{7} = \frac{5^{3}.5^{2}}{7} = \frac{6\times4}{7} = \frac{24}{7} = 3$ Ans

$$\left.\begin{array}{l} \frac{5^1}{7} = 5 \\ \frac{5^2}{7} = 4 \\ \frac{5^3}{7} = 6_{(-1)} \end{array}\right\} \text{Cycle-6}$$

Maths by Puneet chaudhary sir

Thanks..
Best of luck guys..